Lecture 7

2025/2026

Microwave Devices and Circuits for Radiocommunications

2025/2026

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
  - Tuesday **12-14**, **P2**
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 24.02.2026 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - **all materials/equipments authorized**

2025/2026

- Laboratory – **associate professor Radu Damian**
  - Monday 14-16, II.13 / (even weeks)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 24.02.2026)
    - personal homework

General theory

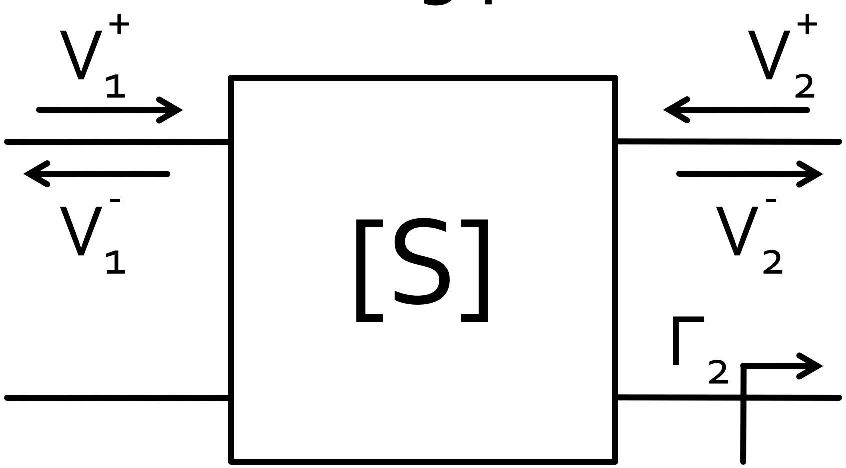
Microwave Network Analysis

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

Scattering matrix – S

- Scattering parameters



$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0} \qquad S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0}$$

- $V_2^+ = 0$ meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

Power waves for N ports

$$[b]=[F]\cdot\left([Z]-[Z_R]^*\right)\cdot\left([Z]+[Z_R]\right)^{-1}\cdot[F]^{-1}\cdot[a]$$

- The scattering matrix for power waves, $[S_p]$

$$[b]=\left[S_p\right]\cdot[a]$$

$$\left[S_p\right]=[F]\cdot\left([Z]-[Z_R]^*\right)\cdot\left([Z]+[Z_R]\right)^{-1}\cdot[F]^{-1}$$

- But: $[S]=\left([Z]-[Z_0]\right)\cdot\left([Z]+[Z_0]\right)^{-1}$

- Typically

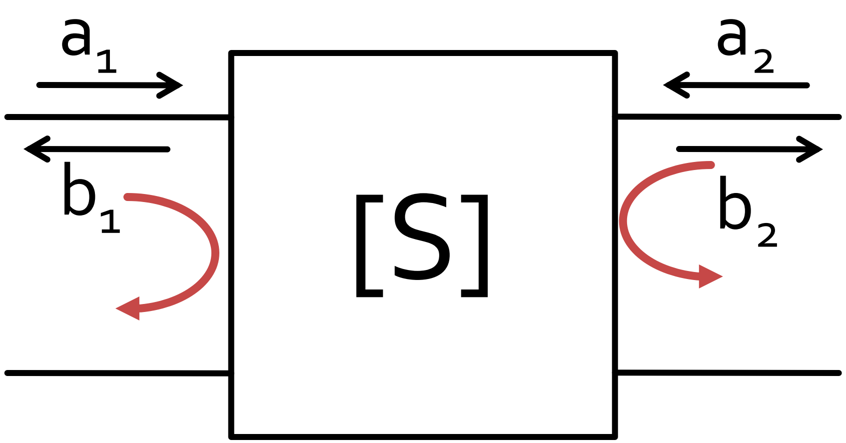
$$Z_{0i}=Z_{Ri}=R_0,\forall i$$

$$R_0=50\Omega$$

$$\left[S_p\right]\equiv[S]$$

- **they coincide!!!**

Scattering matrix – S

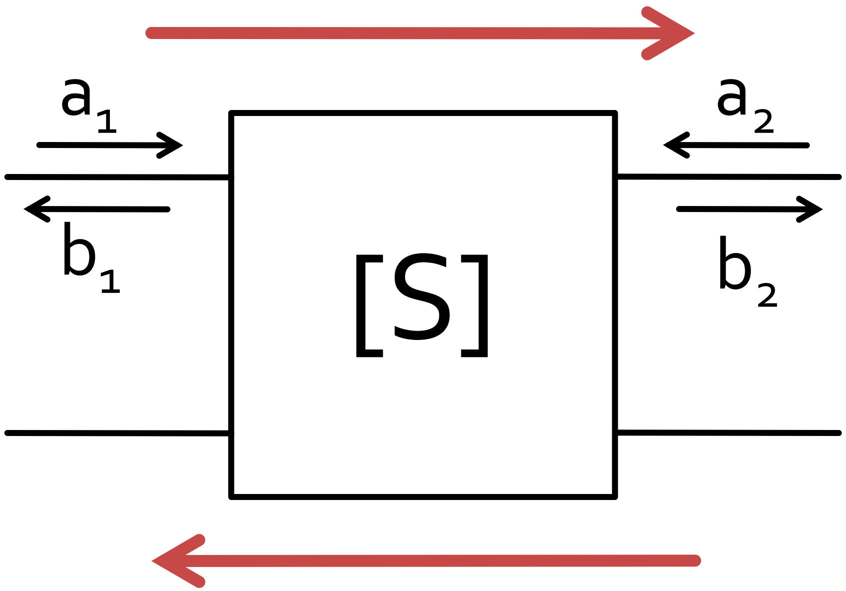


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

- S_{11} and S_{22} are **reflection coefficients** at ports 1 and 2 when the other port is **matched**

Scattering matrix – S

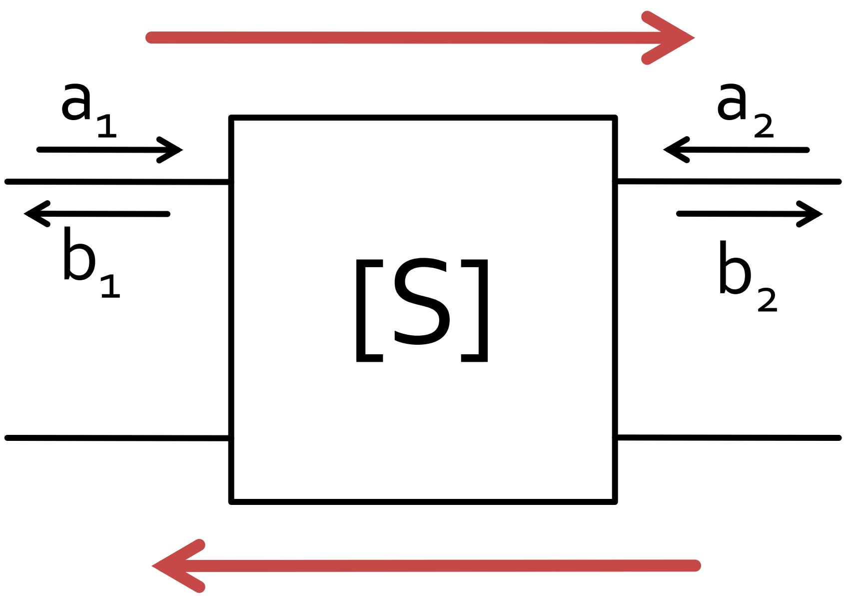


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \qquad S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0}$$

- S_{21} si S_{12} are signal amplitude **gain** when the other port is **matched**

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ from\ Z_0\ source}$$

- a,b
  - information about signal power **AND** signal phase
- S_{ij}
  - network effect (gain) over signal power **including** phase information

Power dividers and directional couplers

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~

Power dividers and couplers

- Desired functionality:
  - division
  - combining
- of signal power

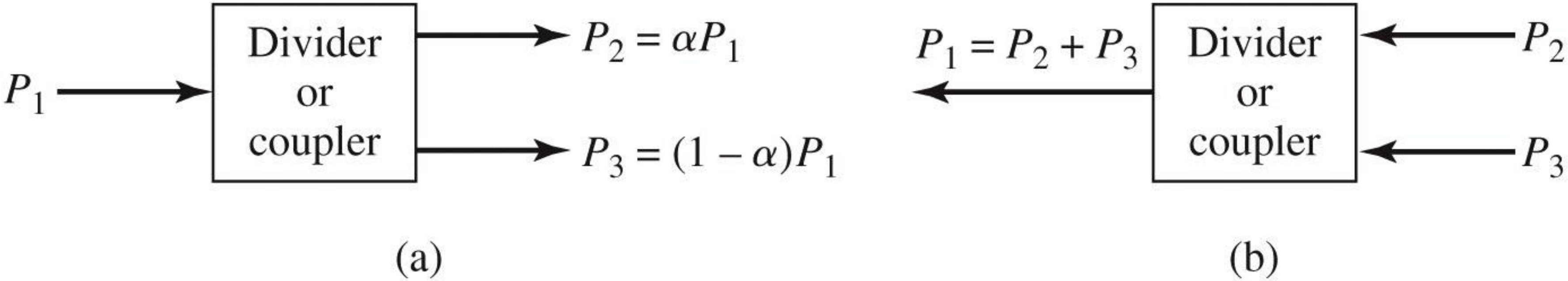


Figure 7.1

Power division of the T-junction

- if the lines are lossless, the network is reciprocal, so it cannot be matched at all ports simultaneously

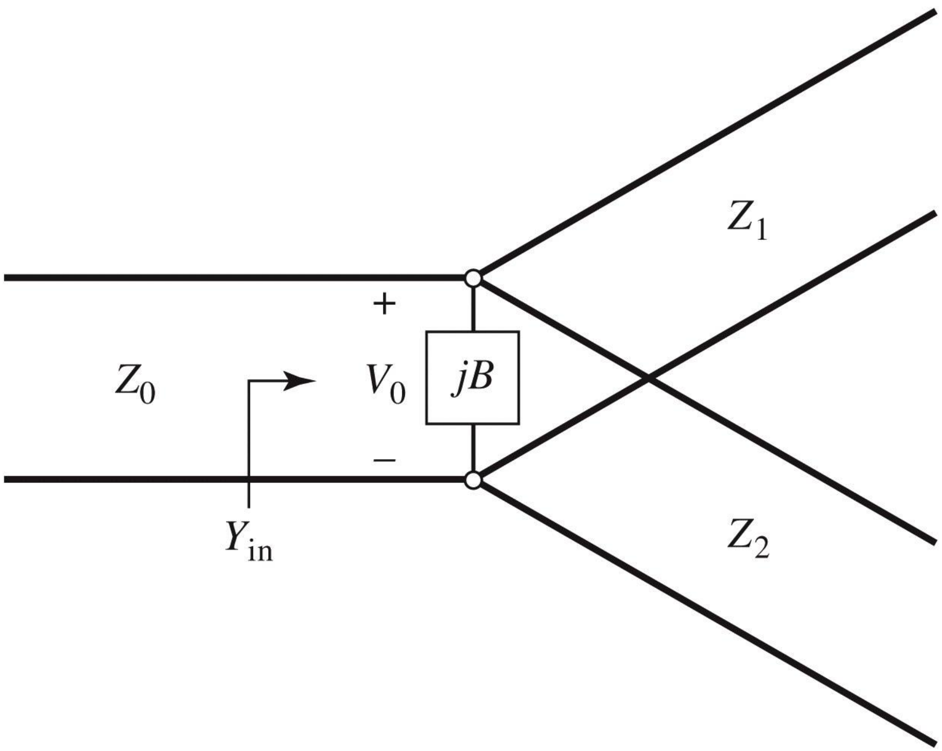


Figure 7.6

- there may be fringing fields and higher order modes associated with the discontinuity at such a junction
- the stored energy can be accounted for by a lumped susceptance: **B**
- Designing the power divider targets matching to the input line Z_o
  - outputs (unmatched, Z_1 and Z_2) can be, if needed, matched to Z_o (λ/4, binomial, Chebyshev)

Resistive Divider

- If a three-port divider contains lossy components, it can be made to be :
  - reciprocal
  - matched at all ports

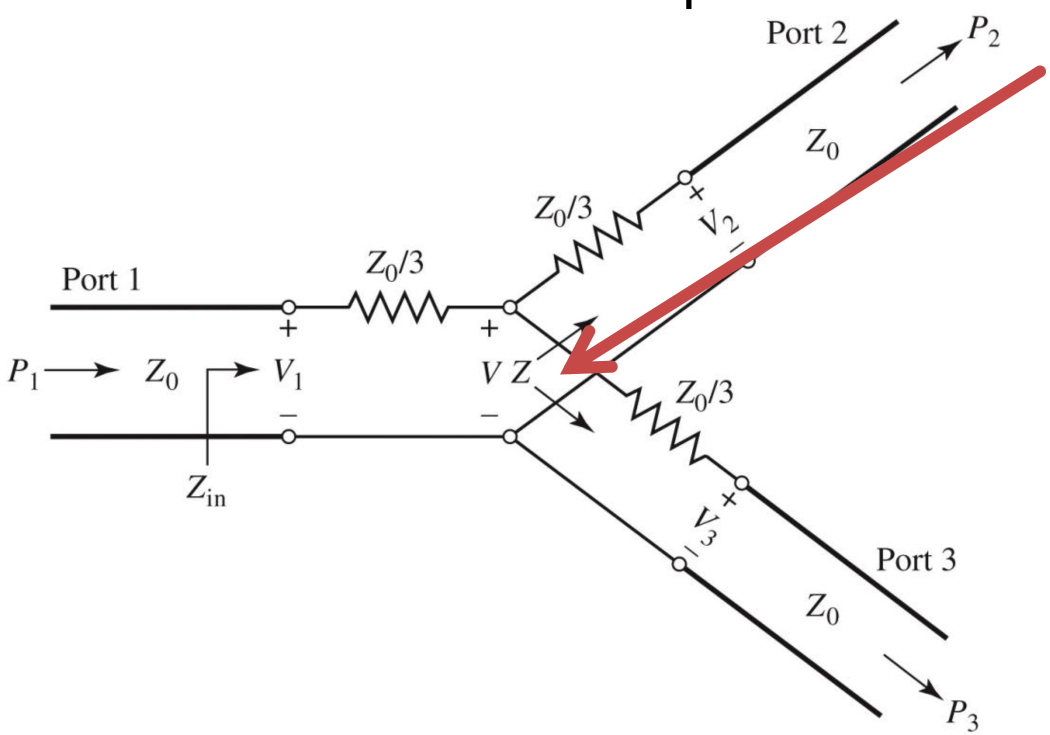


Figure 7.7

The impedance Z, seen looking into the Zo/3 resistor followed by a terminated output line:

$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3}$$

The input line will be terminated with a Zo/3 resistor in series with two such lines Z in parallel

$$Z_{in} = \frac{Z_0}{3} + \frac{1}{2} \cdot \frac{4Z_0}{3} = Z_0$$

so it will be matched: $S_{11} = 0$

from symmetry: $S_{11} = S_{22} = S_{33} = 0$

The Wilkinson power divider

- one input line
- two λ/4 transformers
- one resistor between the output lines

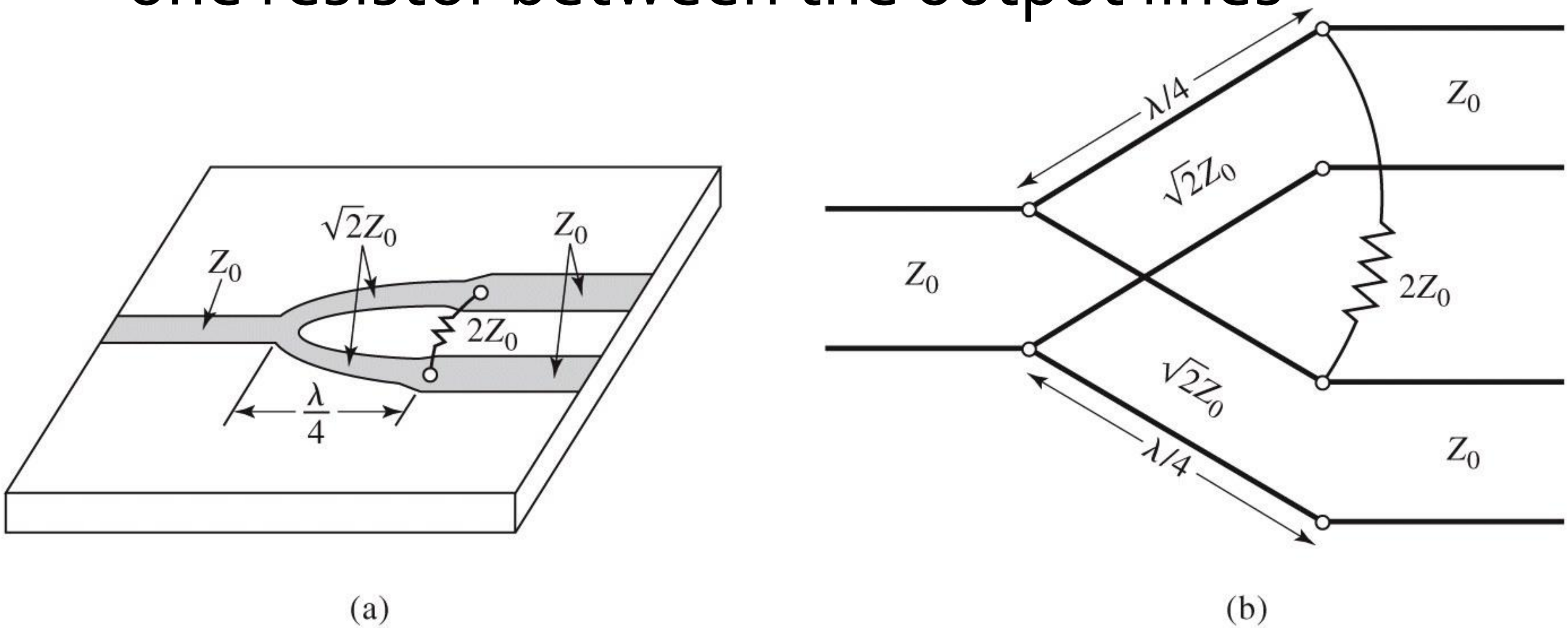
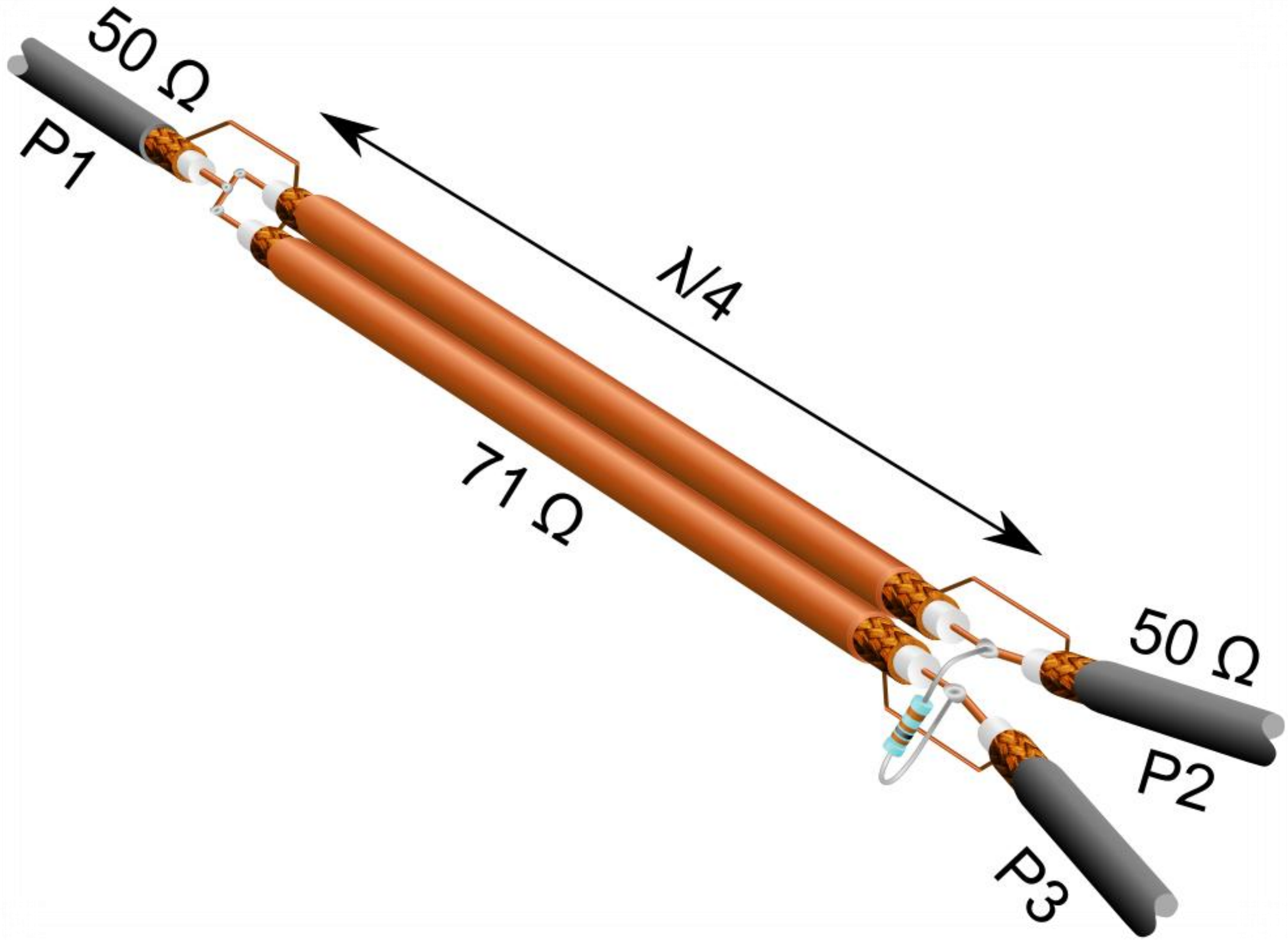


Figure 7.8

The Wilkinson power divider



Directional couplers

Directional Coupler

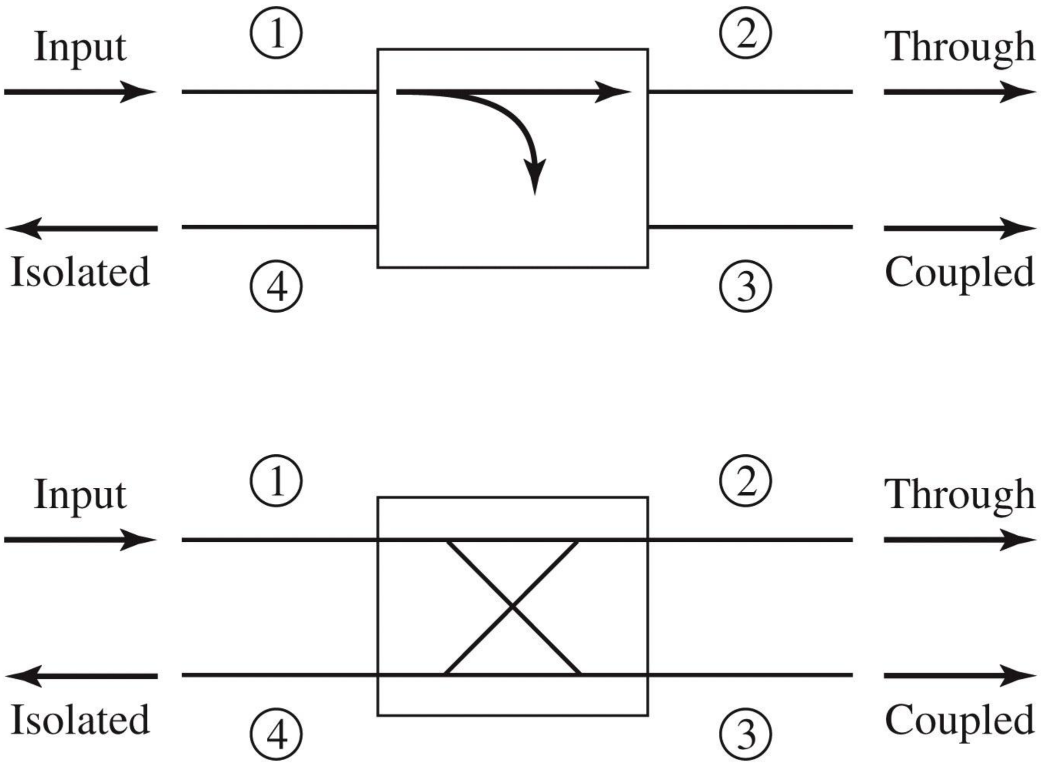


Figure 7.4

$$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$$

$$|S_{13}|^2 = \beta^2$$

Coupling

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\mathrm{dB}]$$

Directivity

$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{|S_{14}|}\right)[\mathrm{dB}]$$

Isolation

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log|S_{14}|\,[\mathrm{dB}]$$

$$I = D + C\,,\ \left[dB\right]$$

Directional Couplers

Laboratory no. 2

The cuadrature (90°) hybrid

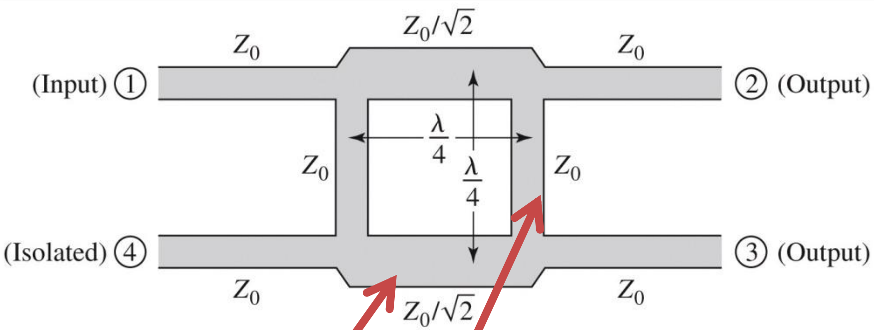


Figure 7.21

$$y_2^2 = 1 + y_1^2$$

$$|\beta| = \frac{\sqrt{y_2^2 - 1}}{y_2}$$

$$C[\mathrm{dB}] = -20 \cdot \log_{10} \frac{\sqrt{y_2^2 - 1}}{y_2}$$

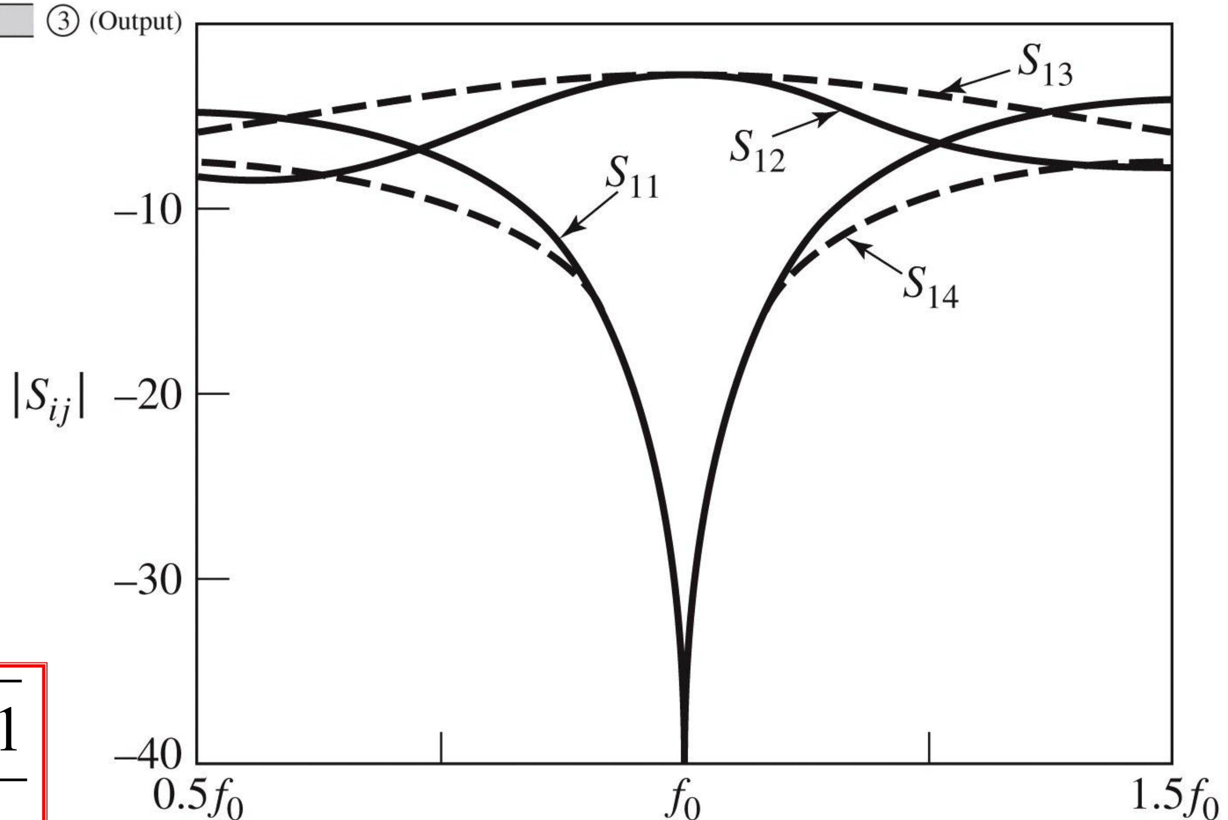
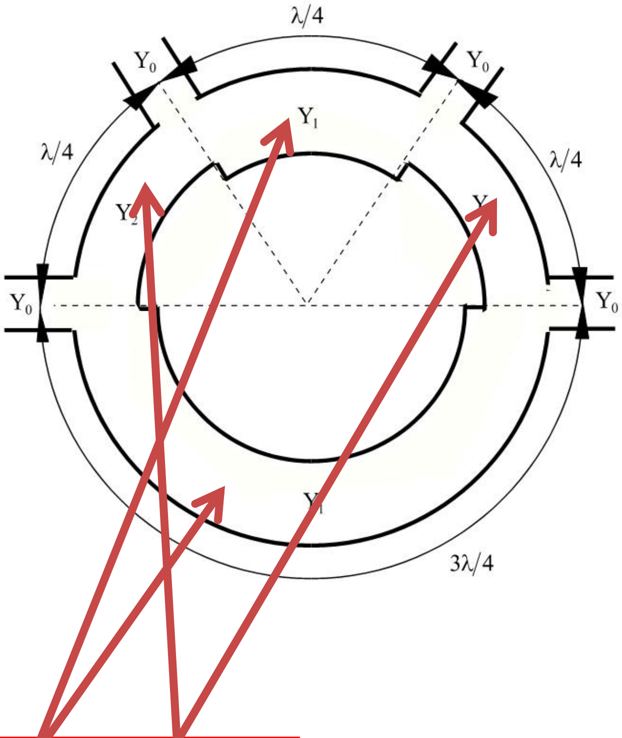


Figure 7.25

The 180° ring hybrid



$$y_1^2 + y_2^2 = 1$$

$$|\beta| = y_1$$

$$C\,[\mathrm{dB}] = -20 \cdot \log_{10}(y_1)$$

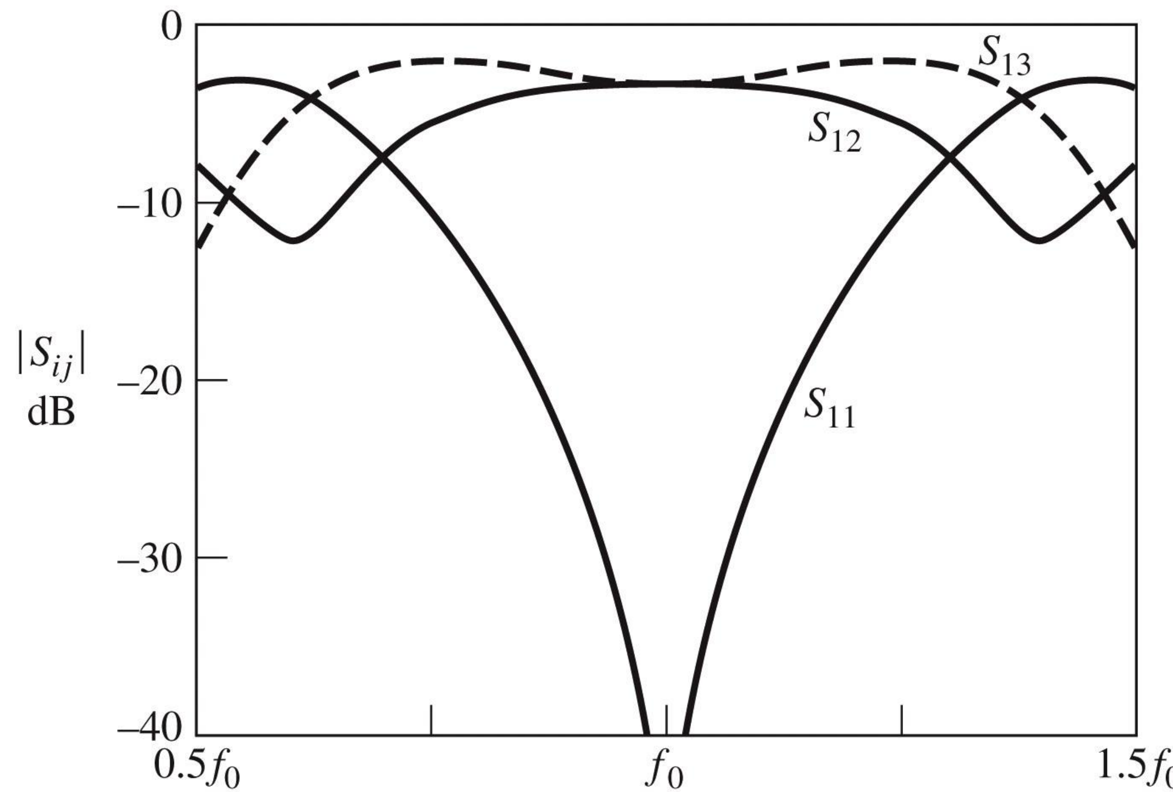
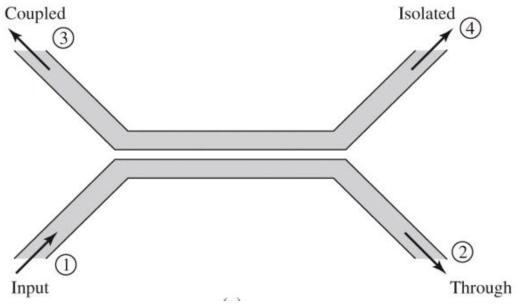


Figure 7.46

Coupled Line Coupler



$$Z_{ce}Z_{co} = Z_0^2$$

$$|\beta| = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$C\,[\mathrm{dB}] = -20 \cdot \log_{10}\left(\frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}\right)$$

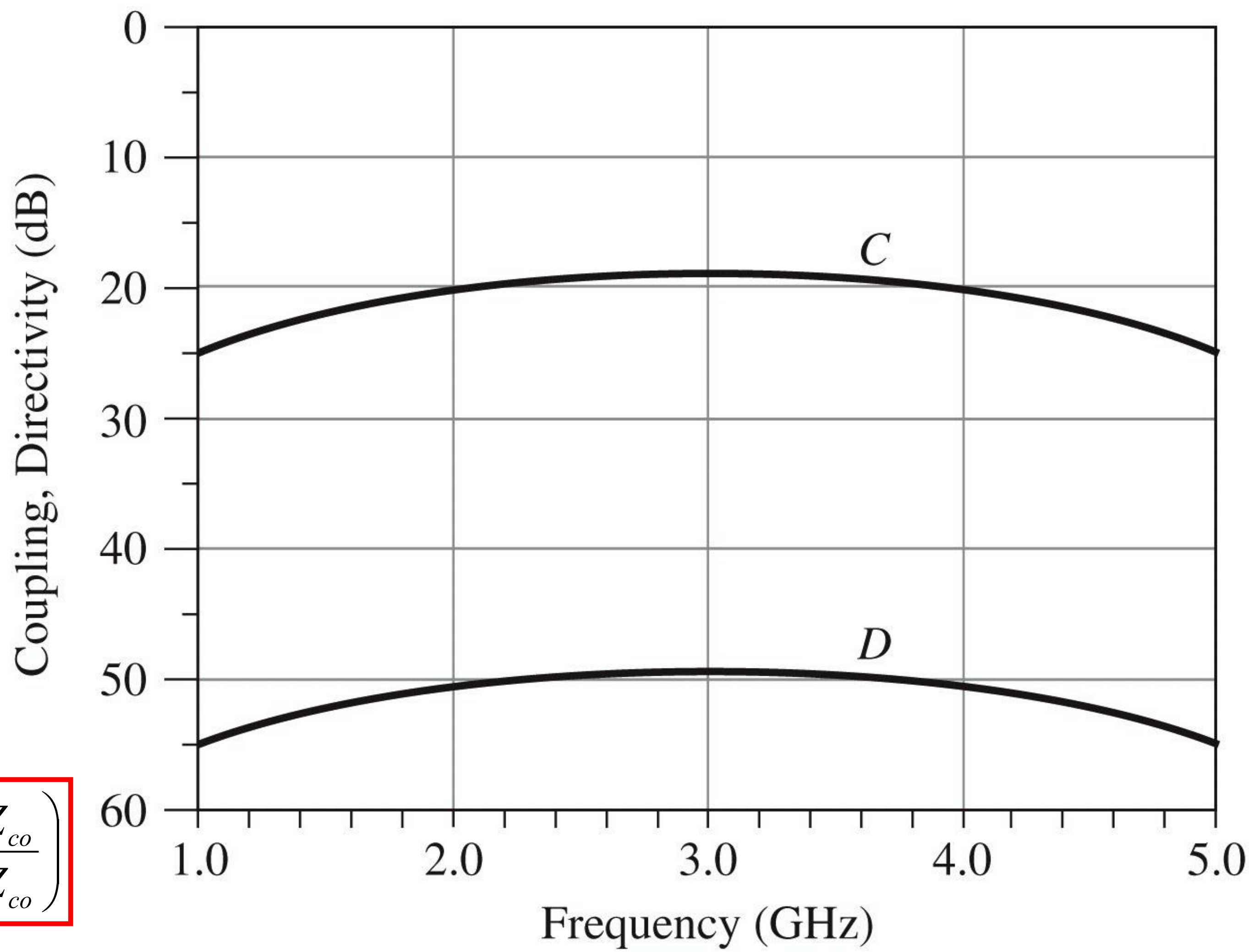


Figure 7.34

Impedance Matching

The Smith Chart

Course Topics

- Transmission lines
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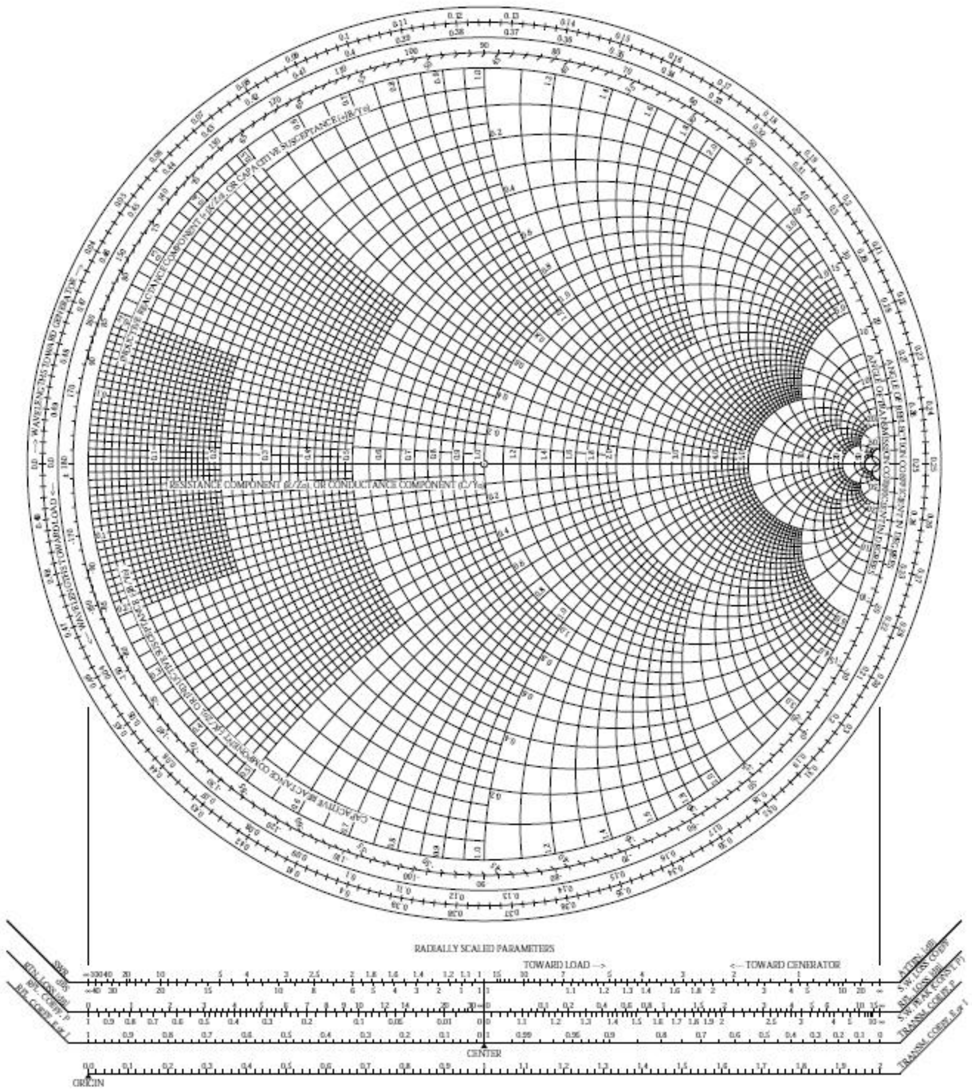
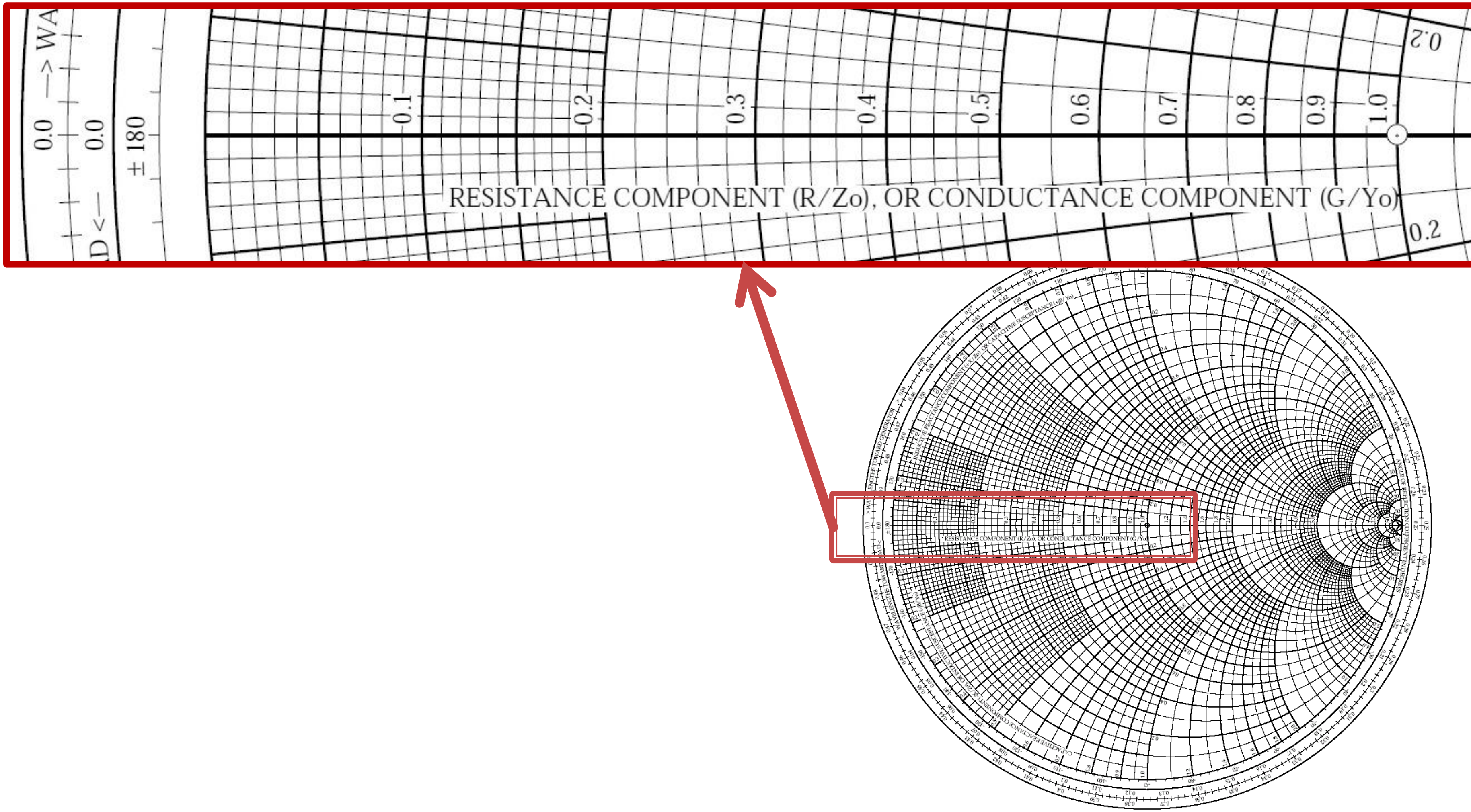
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

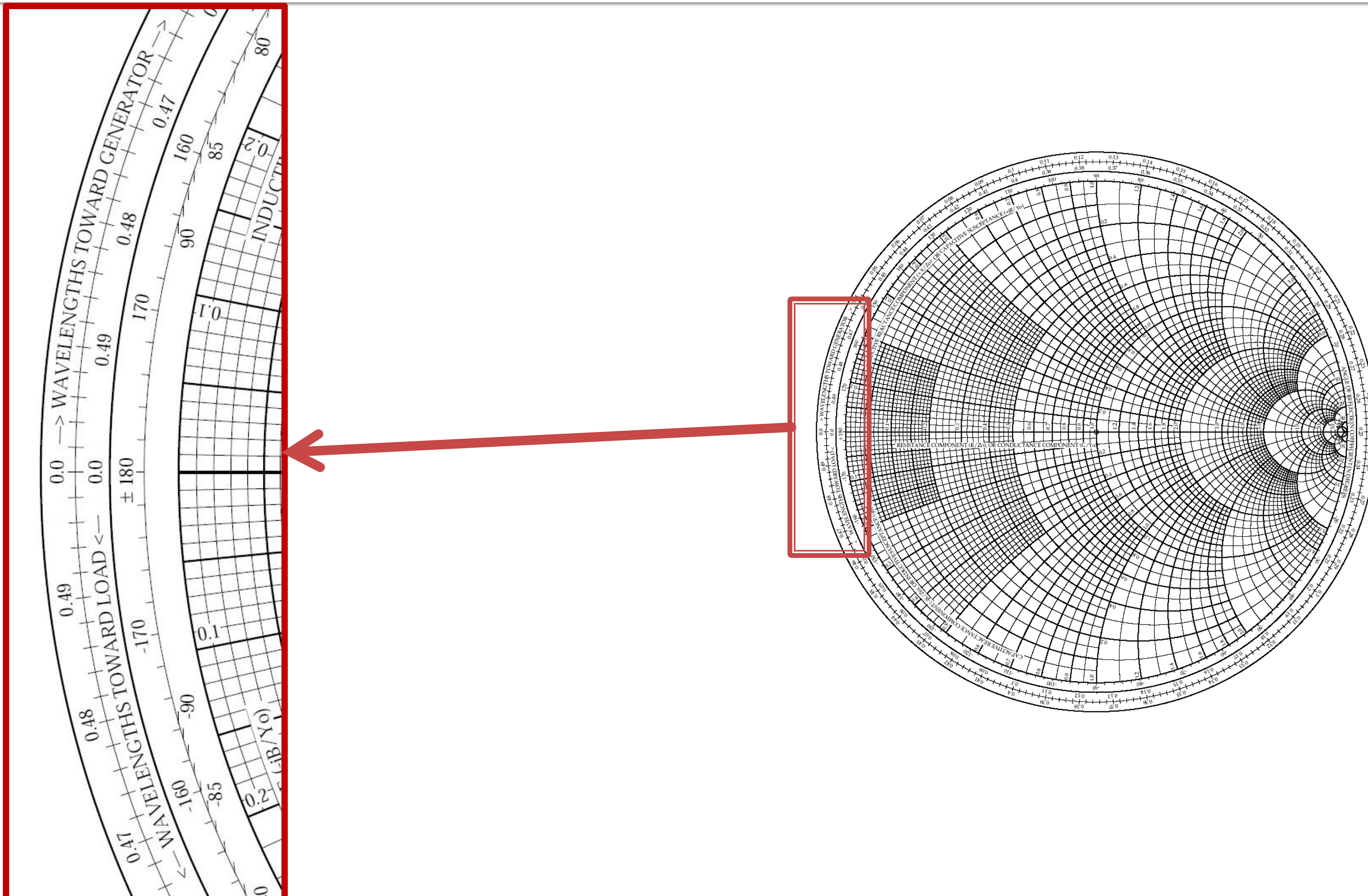
The Smith Chart



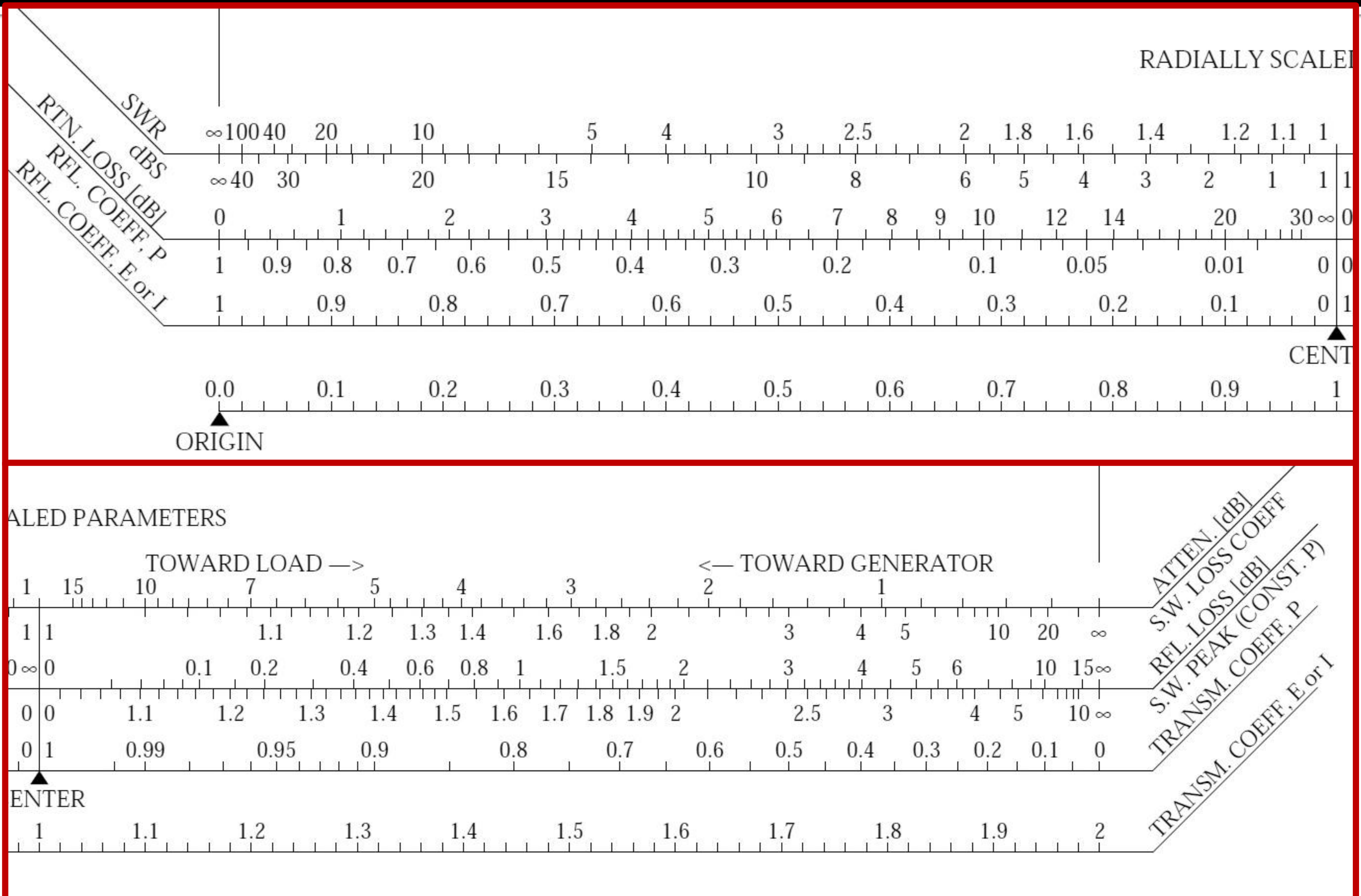
The Smith Chart



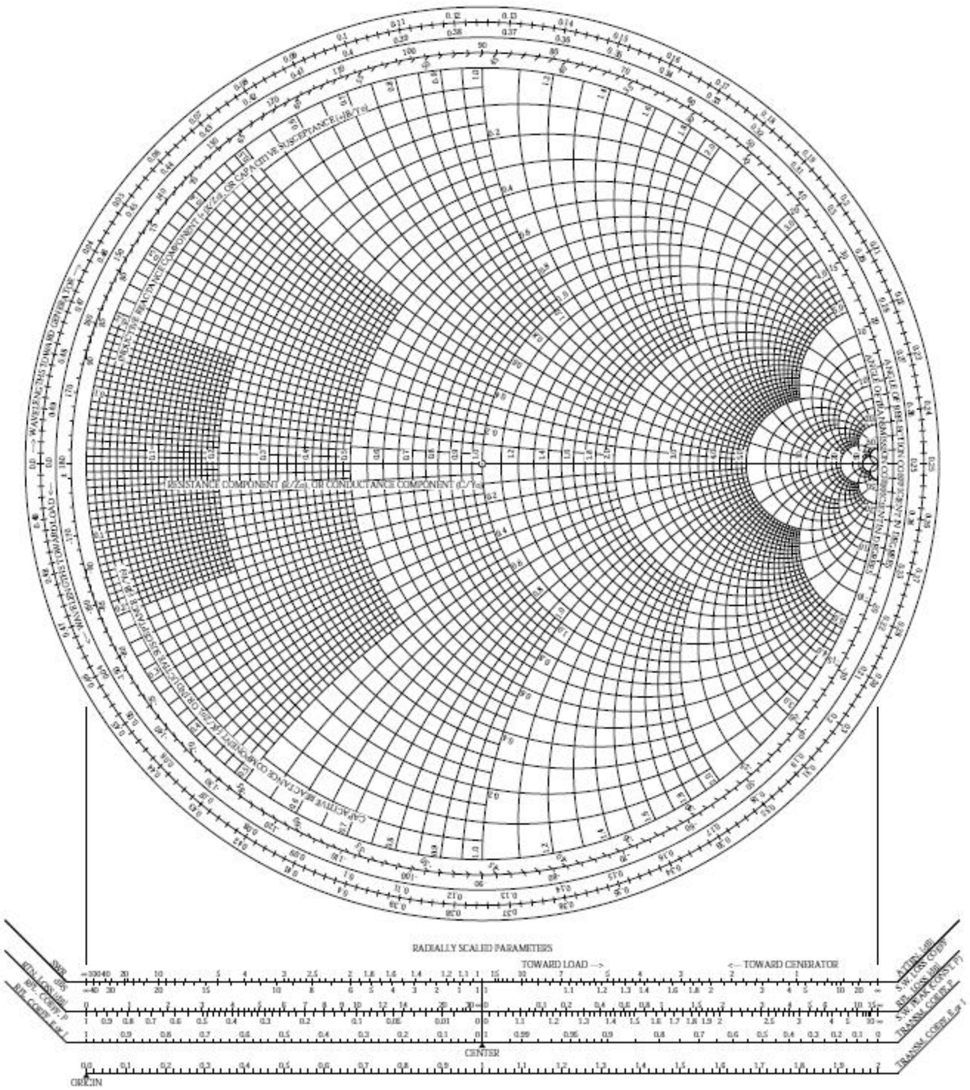
The Smith Chart



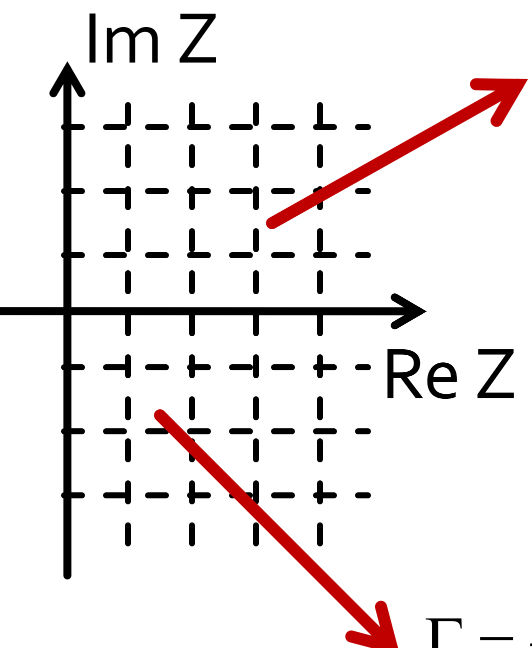
The Smith Chart



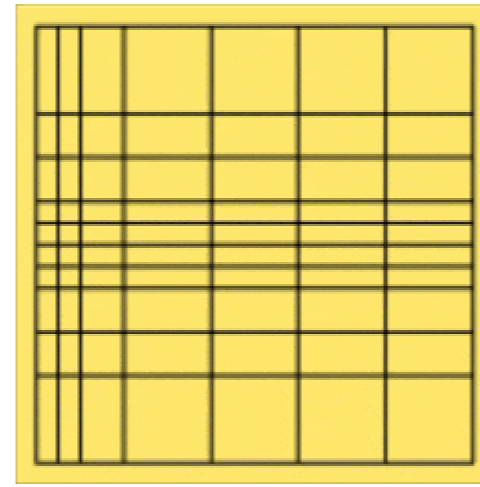
The Smith Chart



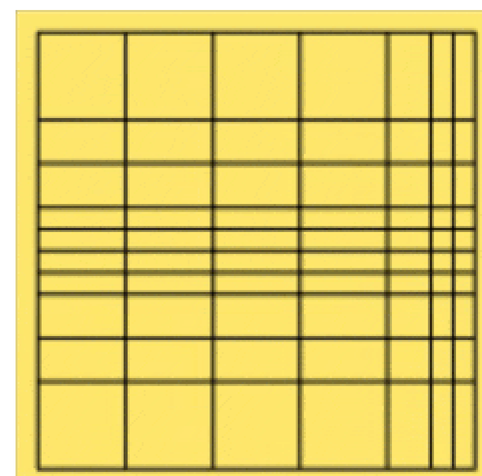
The Smith Chart



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

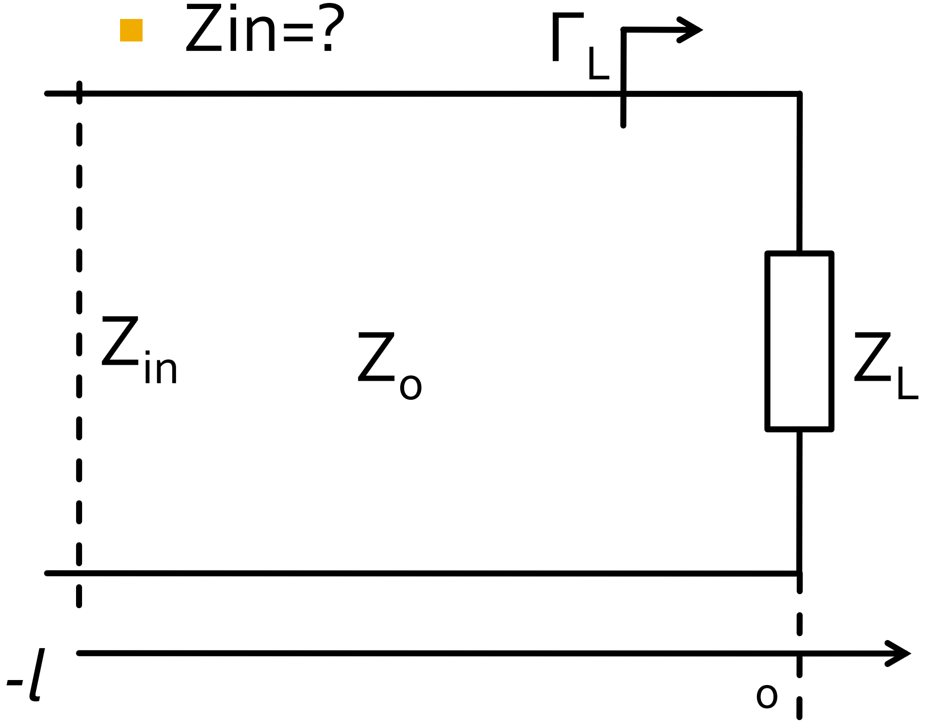


$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_0 - Y_L}{Y_0 + Y_L} = \frac{1 - y_L}{1 + y_L}$$



Traditional usage

- transmission line
  - 100Ω characteristic impedance
  - 0.3λ length
  - Z_L = 40Ω+j·70Ω load
- Zin=?



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$Z_{in} = 36.5340\,\Omega - j \cdot 61.1190\,\Omega$$

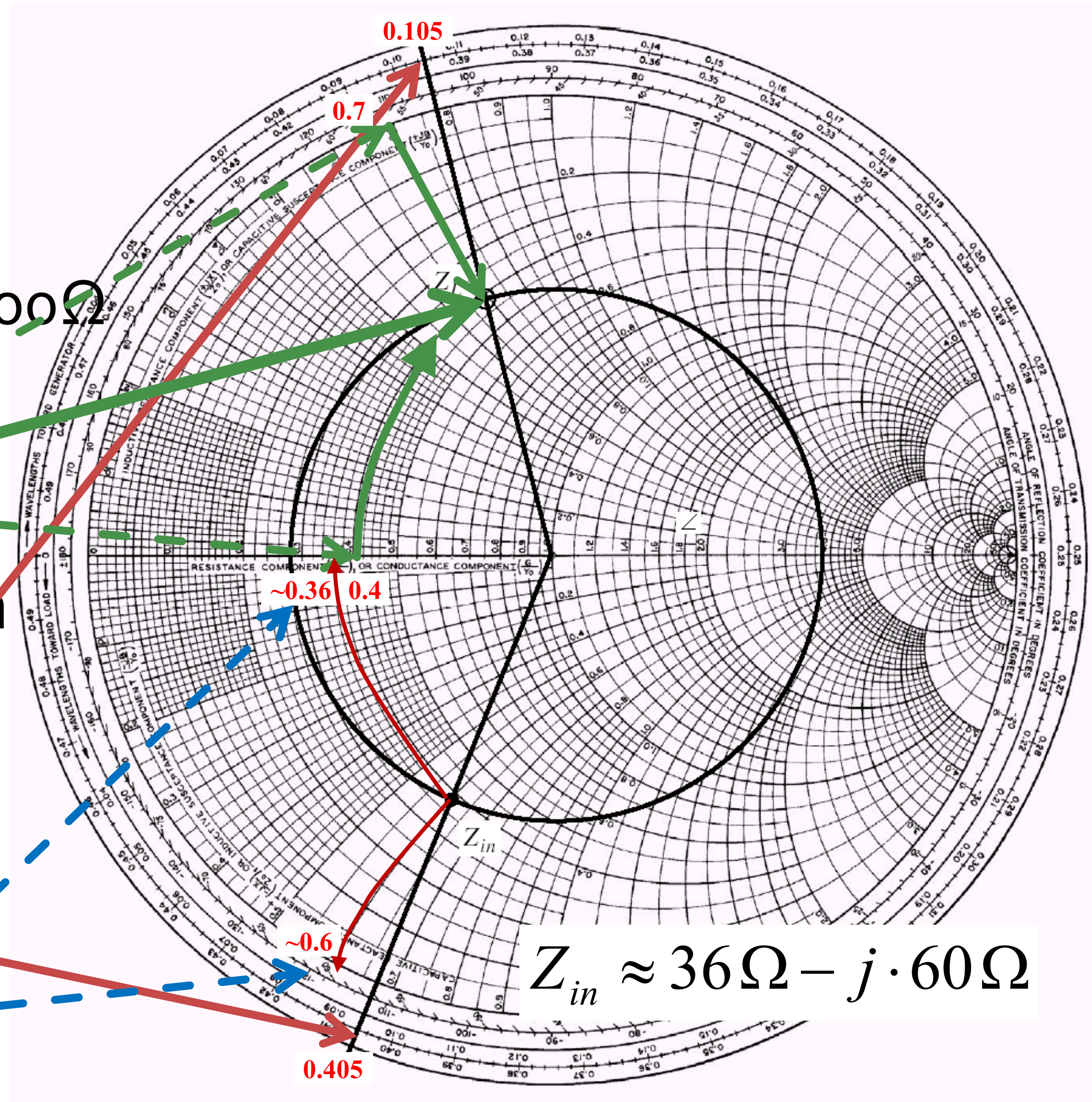
Traditional usage

- transmission line
  - 100Ω impedance
  - 0.3λ length
  - Z_L = 40Ω+j·70Ω load
- normalization with Z_o = 100Ω

$$z_L = \frac{Z_L}{Z_0} = 0.4 + j \cdot 0.7$$

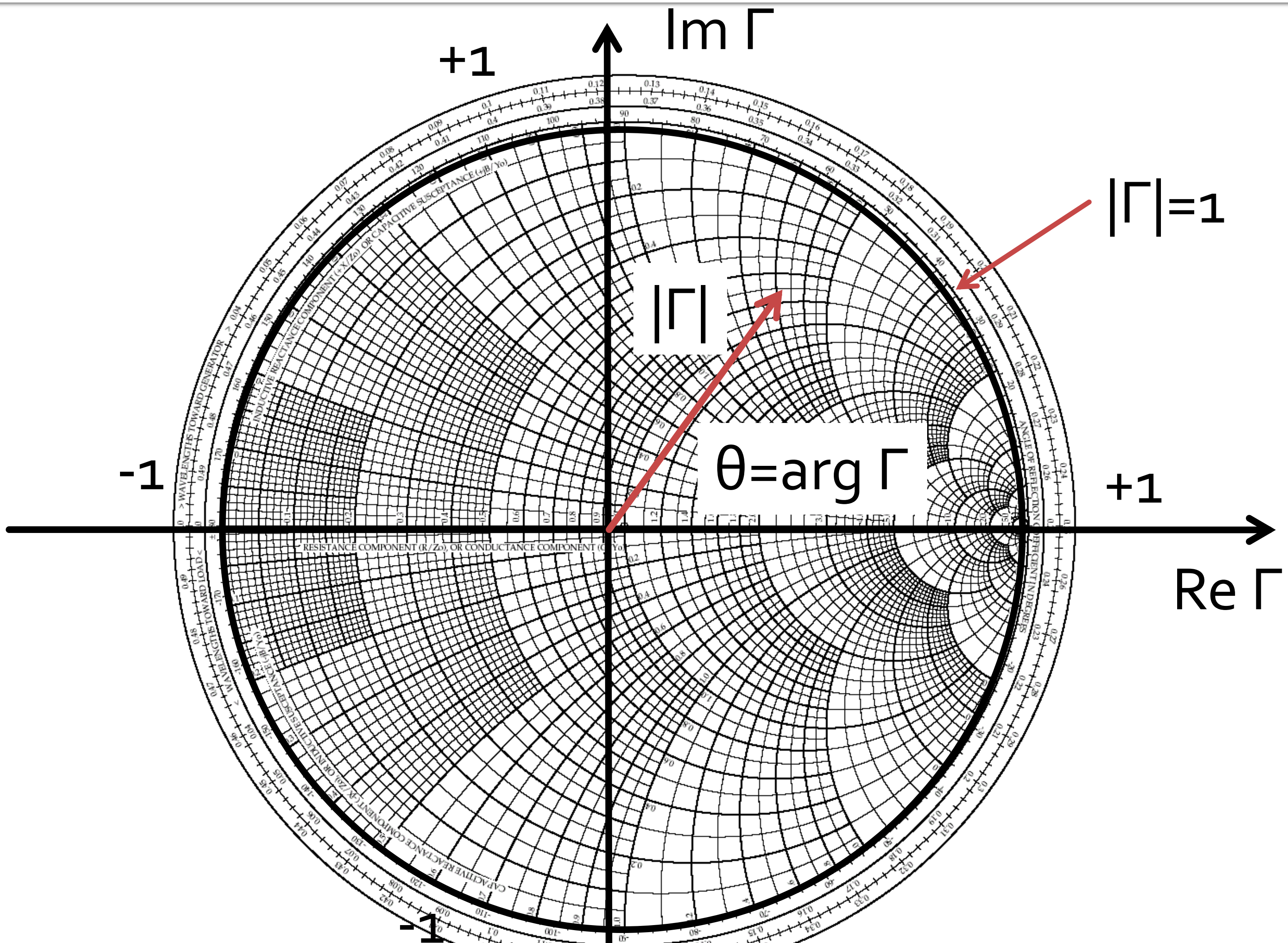
- movement with 0.3λ on a line with Z_o = 100Ω (**circle**)
  - from z_L (0.105λ)
  - to z_{in} (0.405λ)

$$z_{in} \approx 0.36 - j \cdot 0.6 = \frac{Z_{in}}{Z_0}$$

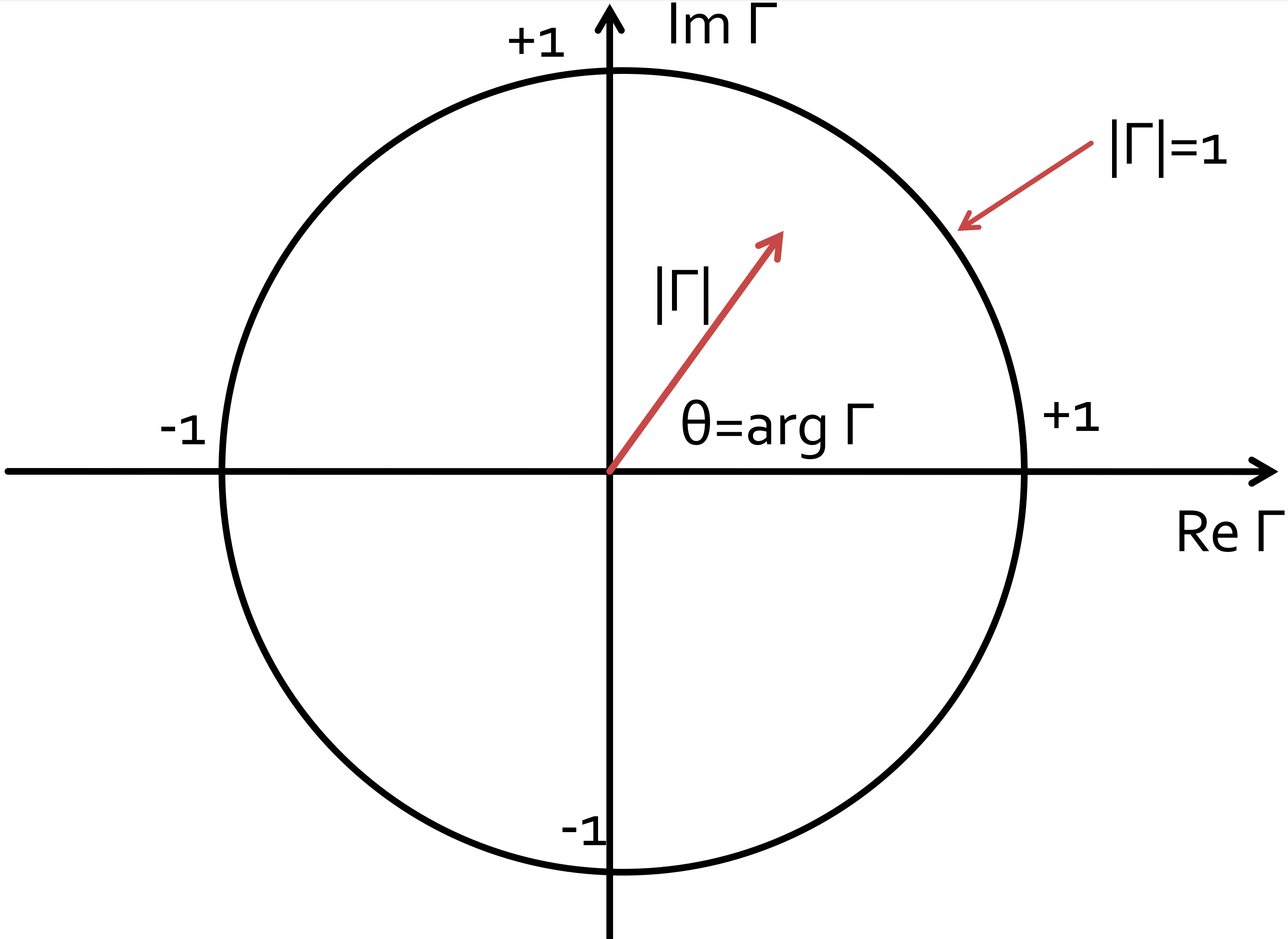


$$Z_{in} \approx 36\,\Omega - j \cdot 60\,\Omega$$

The Smith Chart

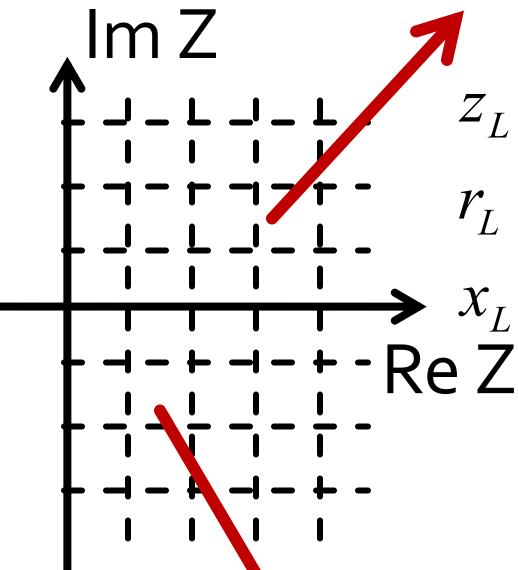


The Smith Chart



The Smith Chart

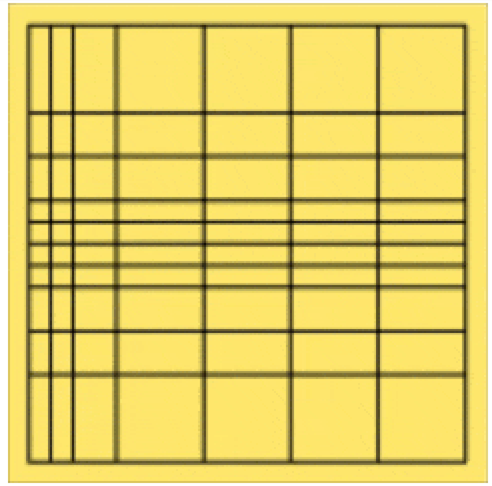
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

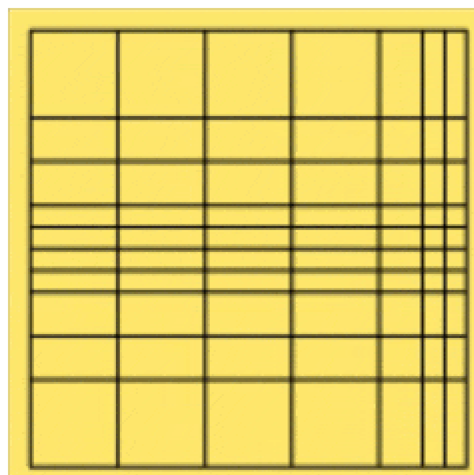


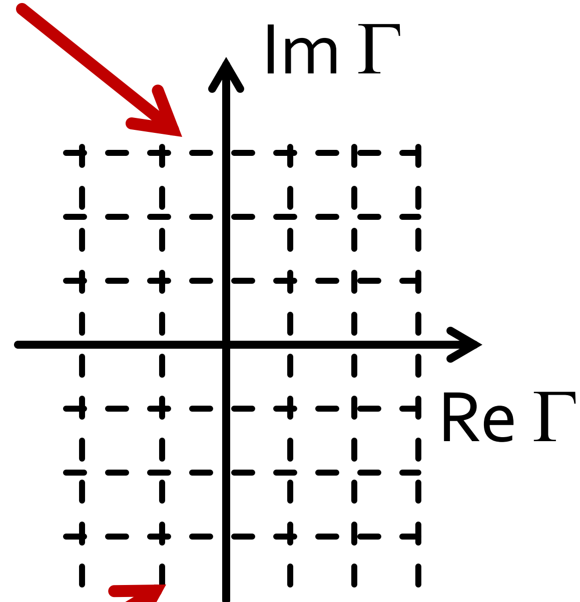
$$z_L = r_L + j \cdot x_L$$

$$r_L = ct.$$

$$x_L = ct.$$

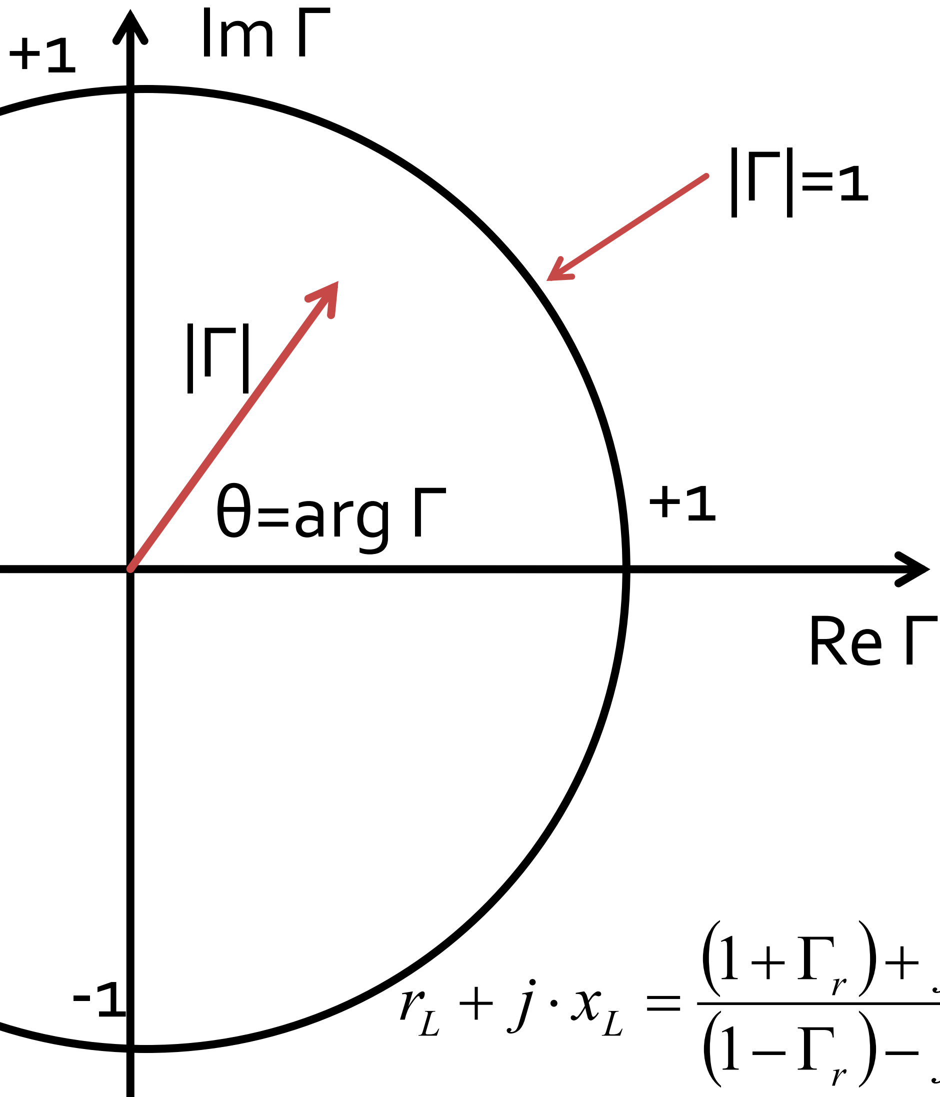






$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_0 - Y_L}{Y_0 + Y_L} = \frac{1 - y_L}{1 + y_L}$$

The Smith Chart



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1} = |\Gamma| \cdot e^{j\theta}$$

$$z_L = \frac{Z_L}{Z_0} \qquad y_L = \frac{Y_L}{Y_0} = \frac{Z_0}{Z_L}$$

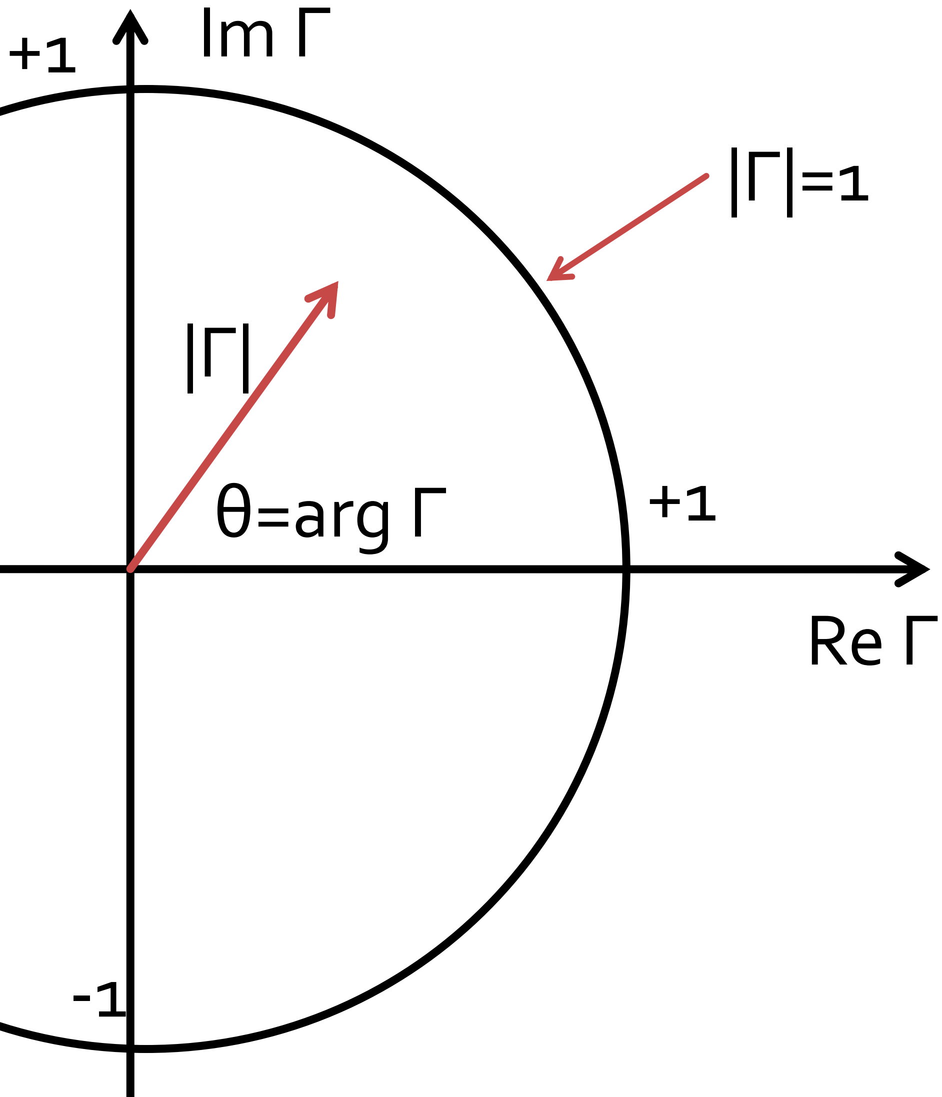
normalization $Z_L \rightarrow z_L$ allows using the same chart for any reference impedance Z_o (the plot becomes independent of the chosen Z_o)

$$\Gamma = \Gamma_r + j \cdot \Gamma_i$$

$$z_L = \frac{1 + |\Gamma| \cdot e^{j\theta}}{1 - |\Gamma| \cdot e^{j\theta}} = r_L + j \cdot x_L$$

$$r_L + j \cdot x_L = \frac{(1 + \Gamma_r) + j \cdot \Gamma_i}{(1 - \Gamma_r) - j \cdot \Gamma_i} = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2} + j \cdot \frac{2 \cdot \Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

The Smith Chart



$$r_L = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}$$

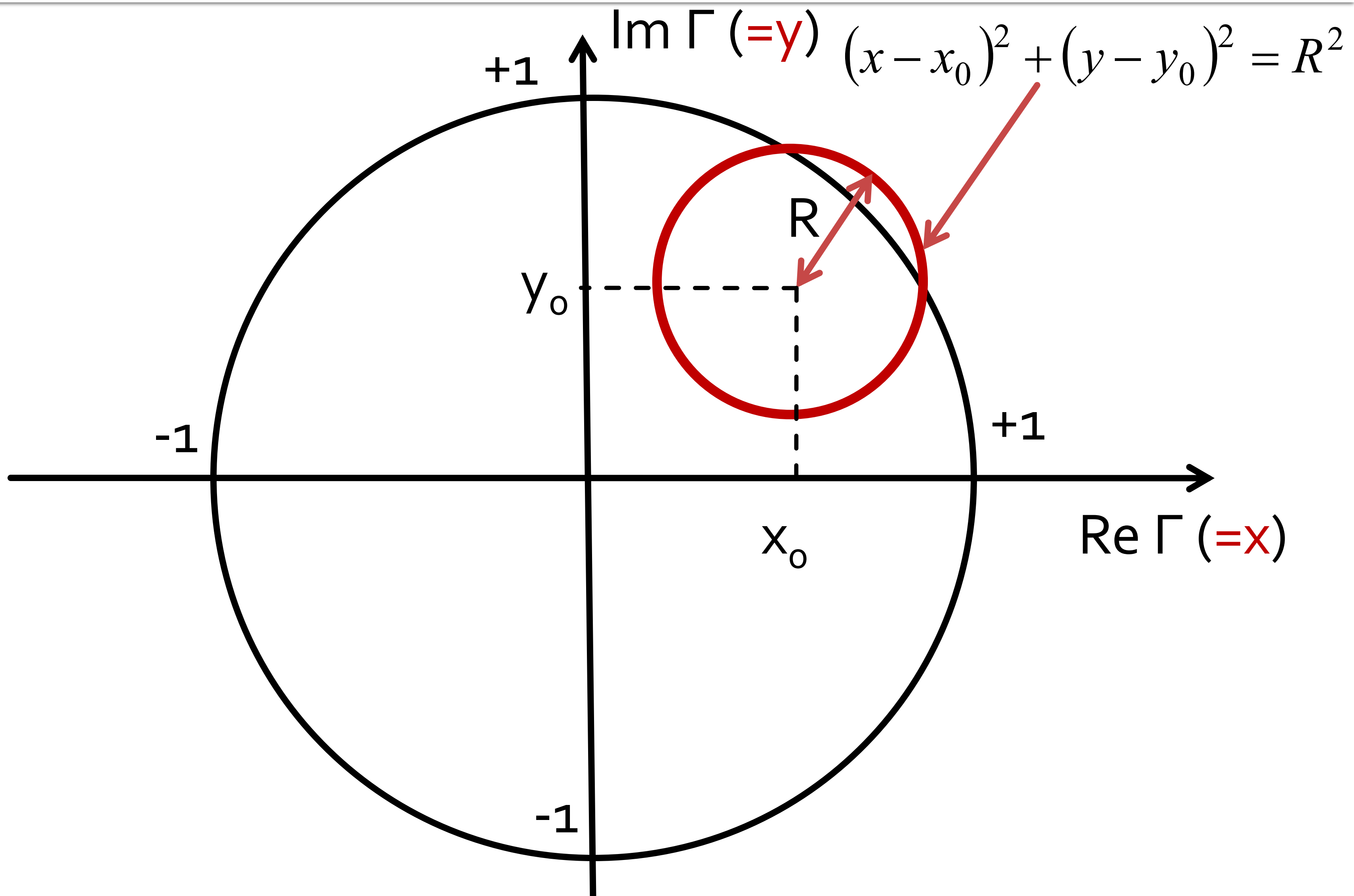
$$x_L = \frac{2\cdot\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2}$$

- Rearranged

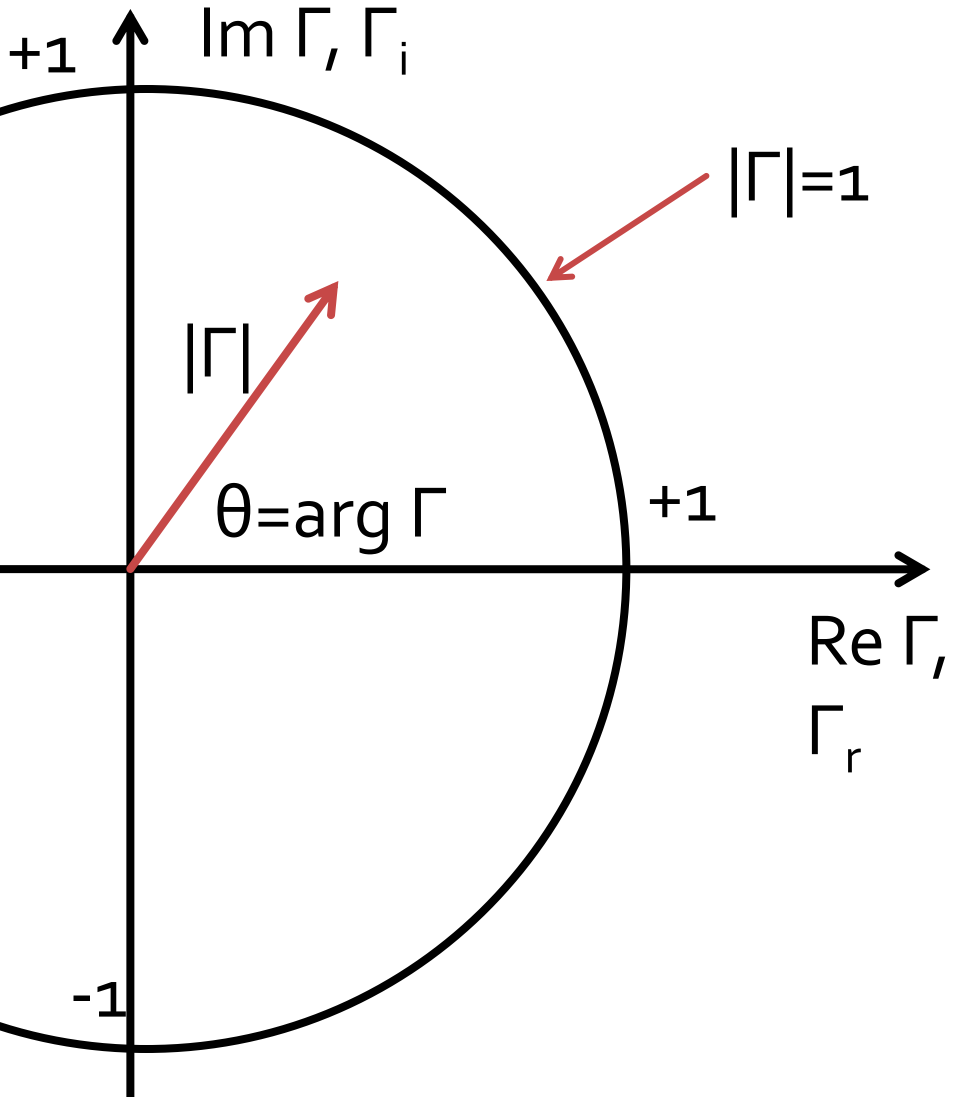
$$\left(\Gamma_r-\frac{r_L}{1+r_L}\right)^2+\Gamma_i^2=\left(\frac{1}{1+r_L}\right)^2$$

$$(\Gamma_r-1)^2+\left(\Gamma_i-\frac{1}{x_L}\right)^2=\left(\frac{1}{x_L}\right)^2$$

The Smith Chart



The Smith Chart



$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2$$

$$\left(\Gamma_r - 1\right)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

- **Circles** in the (Γ_r, Γ_i) complex plane

$$\left(x - x_0\right)^2 + \left(y - y_0\right)^2 = R^2$$

The Smith Chart, resistance

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2$$

$$(x-x_0)^2 + (y-y_0)^2 = R^2$$

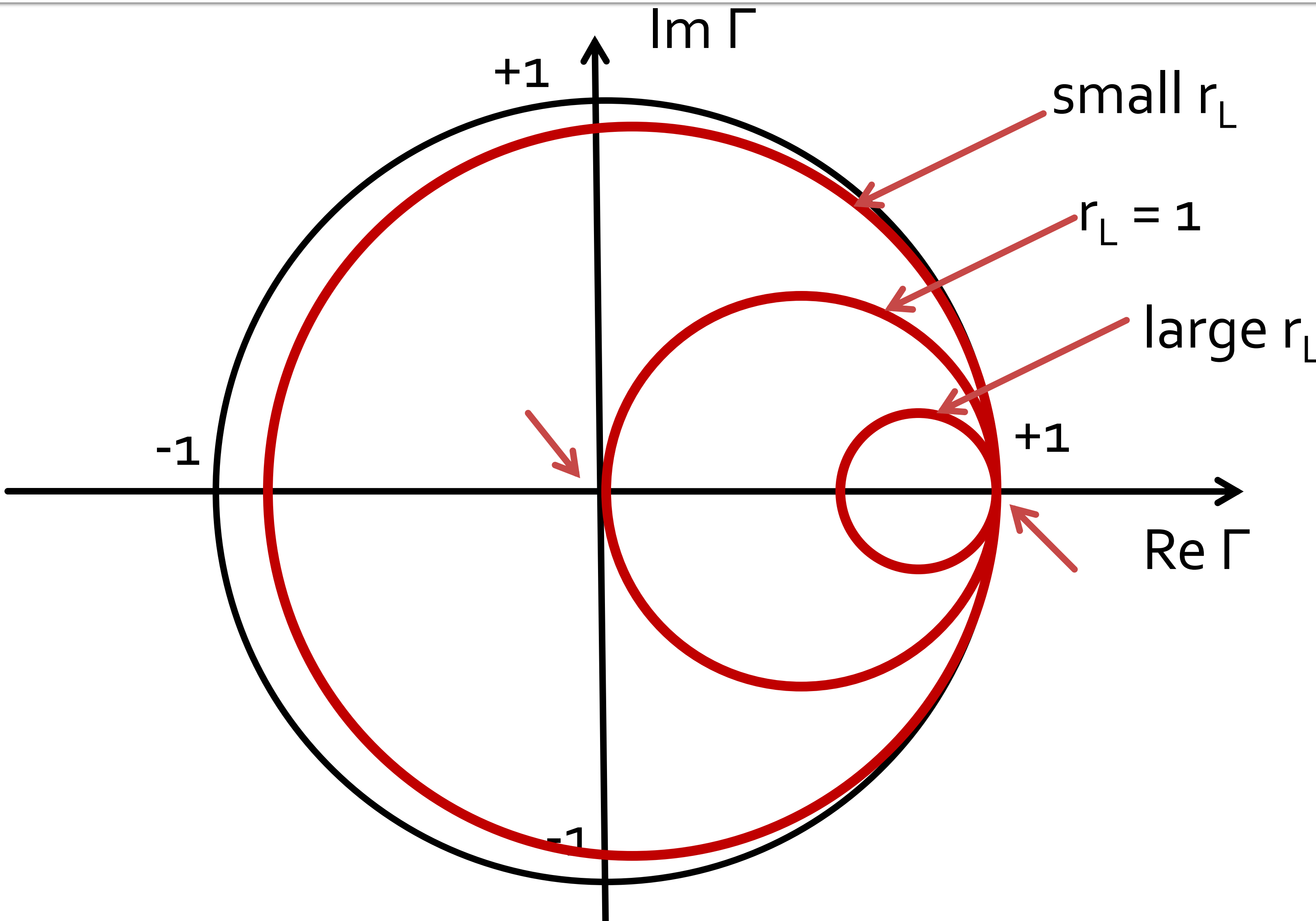
$$\begin{cases} x_0 = \dfrac{r_L}{1+r_L} \\ y_0 = 0 \\ R = \dfrac{1}{1+r_L} \end{cases}$$

- The locus (the set of all points whose location satisfies one or more specified conditions) of the points generated by all impedances having normalized resistance r_L is a circle which:
  - have its **center on the horizontal axis** ($y_0=0$)
  - passes through the point **x=1,y=0**, whatever x_0, r_L
  - have its radius between 0 and 1
    - tends to 0 for large r_L
    - tends to 1 for small r_L
  - when r_L is **1** passes also through **origin**
  - for any **pozitive** r_L radius is **<1**

$$\left(1 - \frac{r_L}{1+r_L}\right)^2 + 0 = \left(\frac{1}{1+r_L}\right)^2$$

$$\left(0 - \frac{r_L}{1+r_L}\right)^2 = \left(\frac{1}{1+r_L}\right)^2 \leftrightarrow r_L = 1$$

The Smith Chart, resistance



The Smith Chart, reactance

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

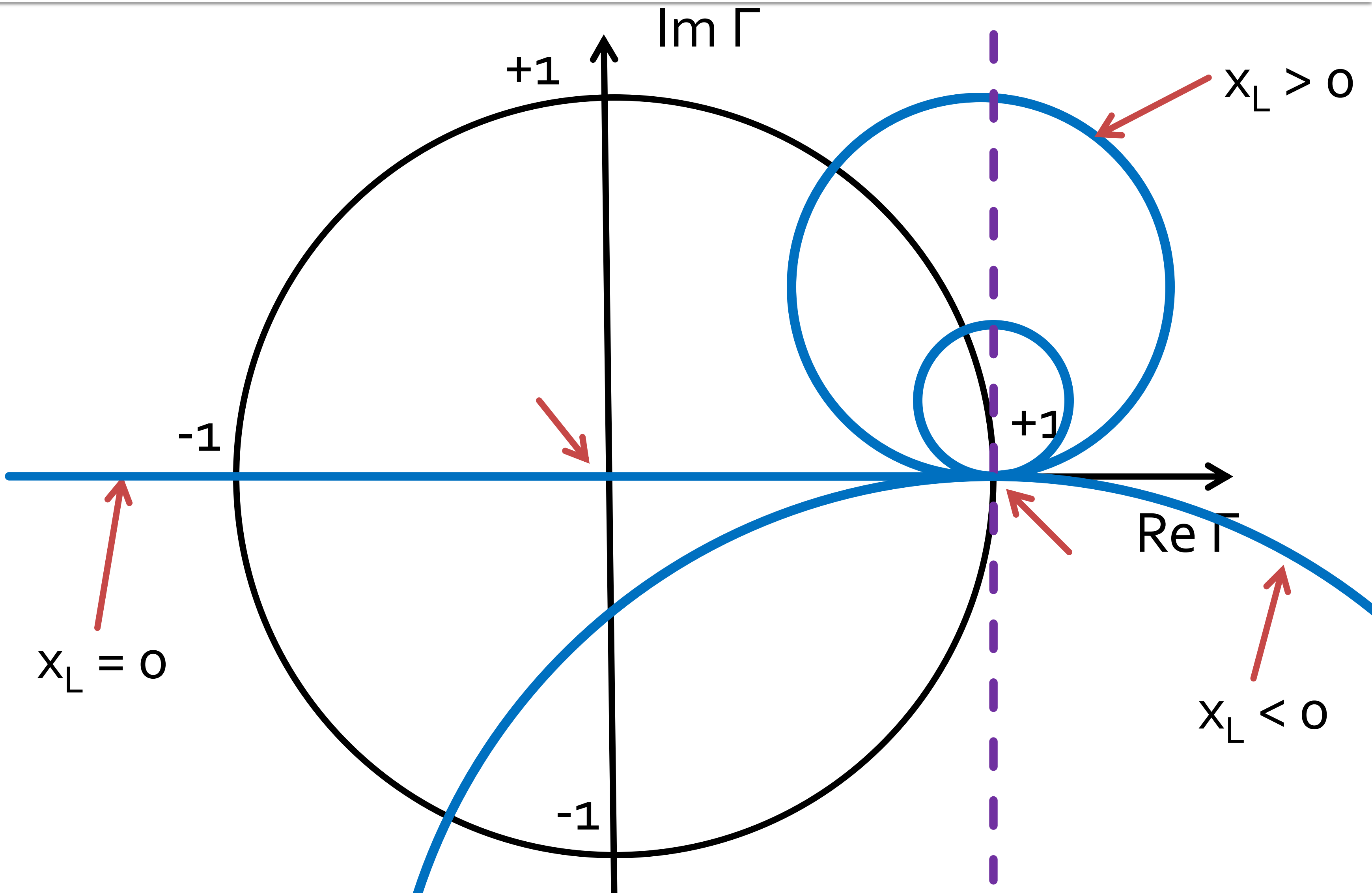
$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

$$\begin{cases} x_0 = 1 \\ y_0 = \dfrac{1}{x_L} \\ R = \dfrac{1}{|x_L|} \end{cases}$$

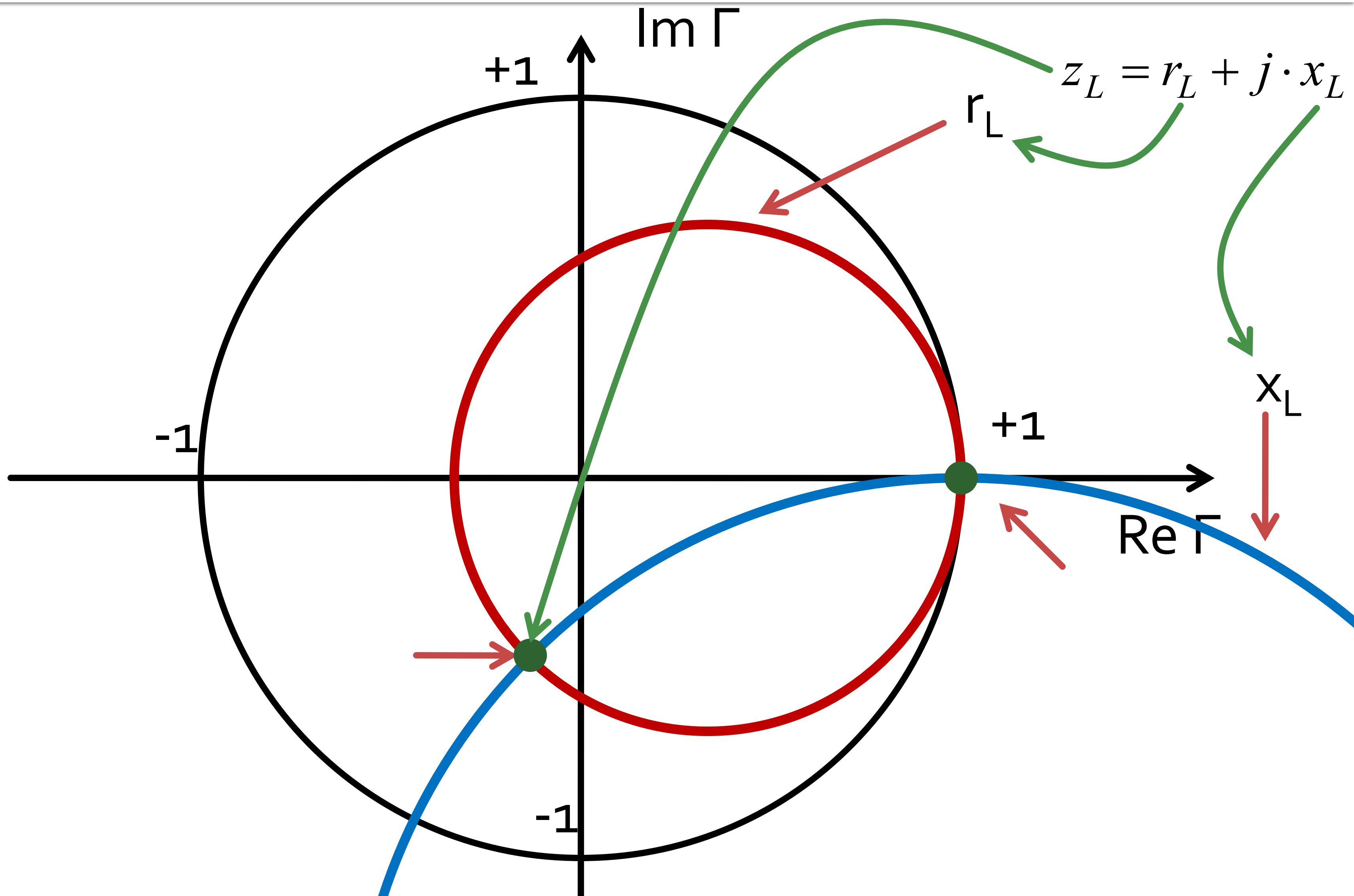
- The locus of the points generated by all impedances having normalized resistance x_L is a circle which:
  - have its **center on a line parallel with the vertical axis** ($x_0=1$)
  - passes through **x=1,y=0** point, whatever x_0,x_L
  - have its radius between 0 and ∞
    - tends to 0 for large $|x_L|$
    - tends to ∞ for small $|x_L|$
  - when x_L is **0** transforms itself in the **horizontal axis**
  - if $x_L > 0$ the circle is above the horizontal axis, otherwise is bellow it

$$(1-1)^2 + \left(0 - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

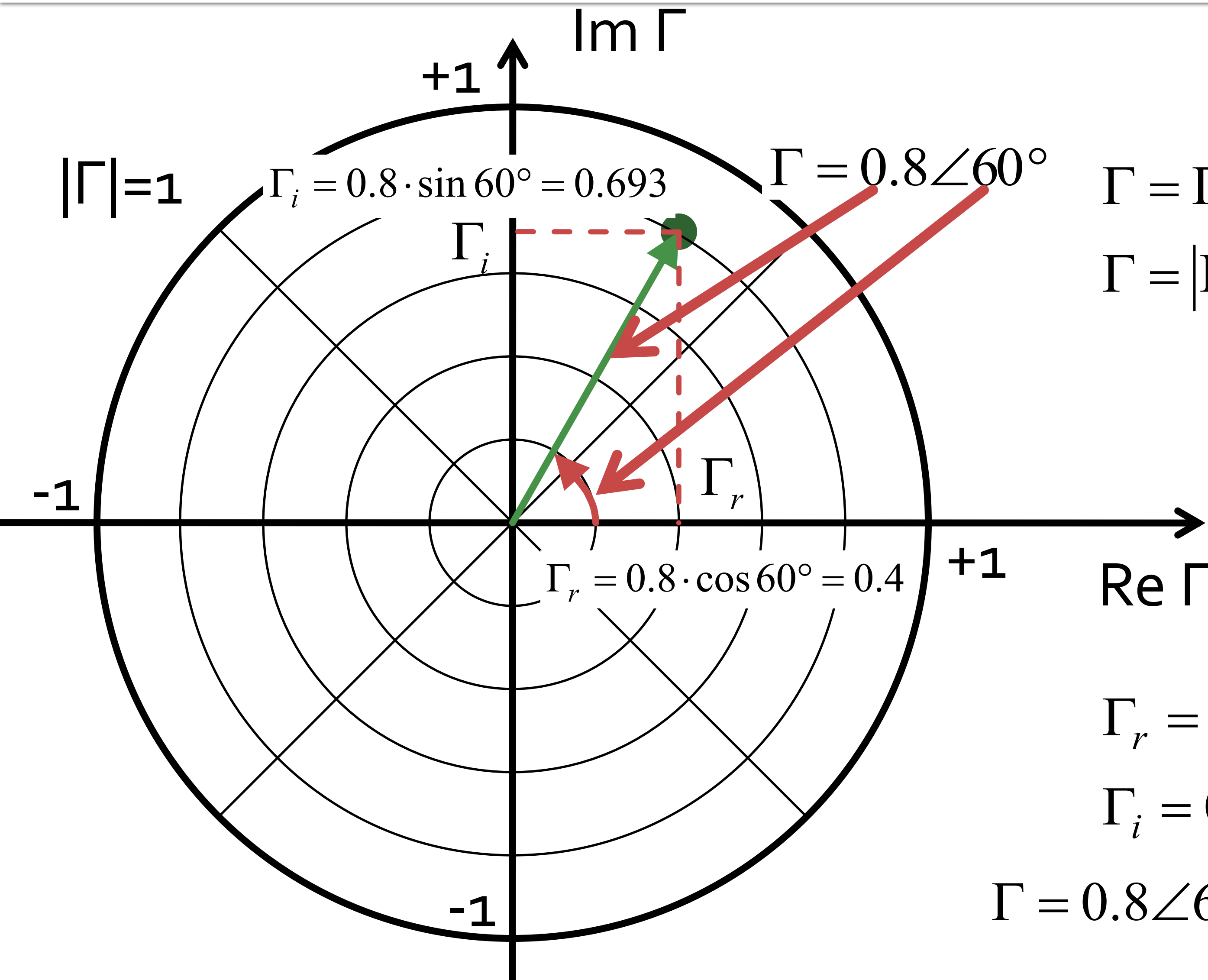
The Smith Chart, reactance



The Smith Chart, impedance



The Smith Chart, reflection coefficient, Cartesian coordinate system



$$\Gamma = \Gamma_r + j \cdot \Gamma_i$$

$$\Gamma = |\Gamma| \cdot (\cos\theta + j \cdot \sin\theta)$$

$$\Gamma = |\Gamma| \cdot e^{j\theta}$$

$$\Gamma = |\Gamma| \angle \theta°$$

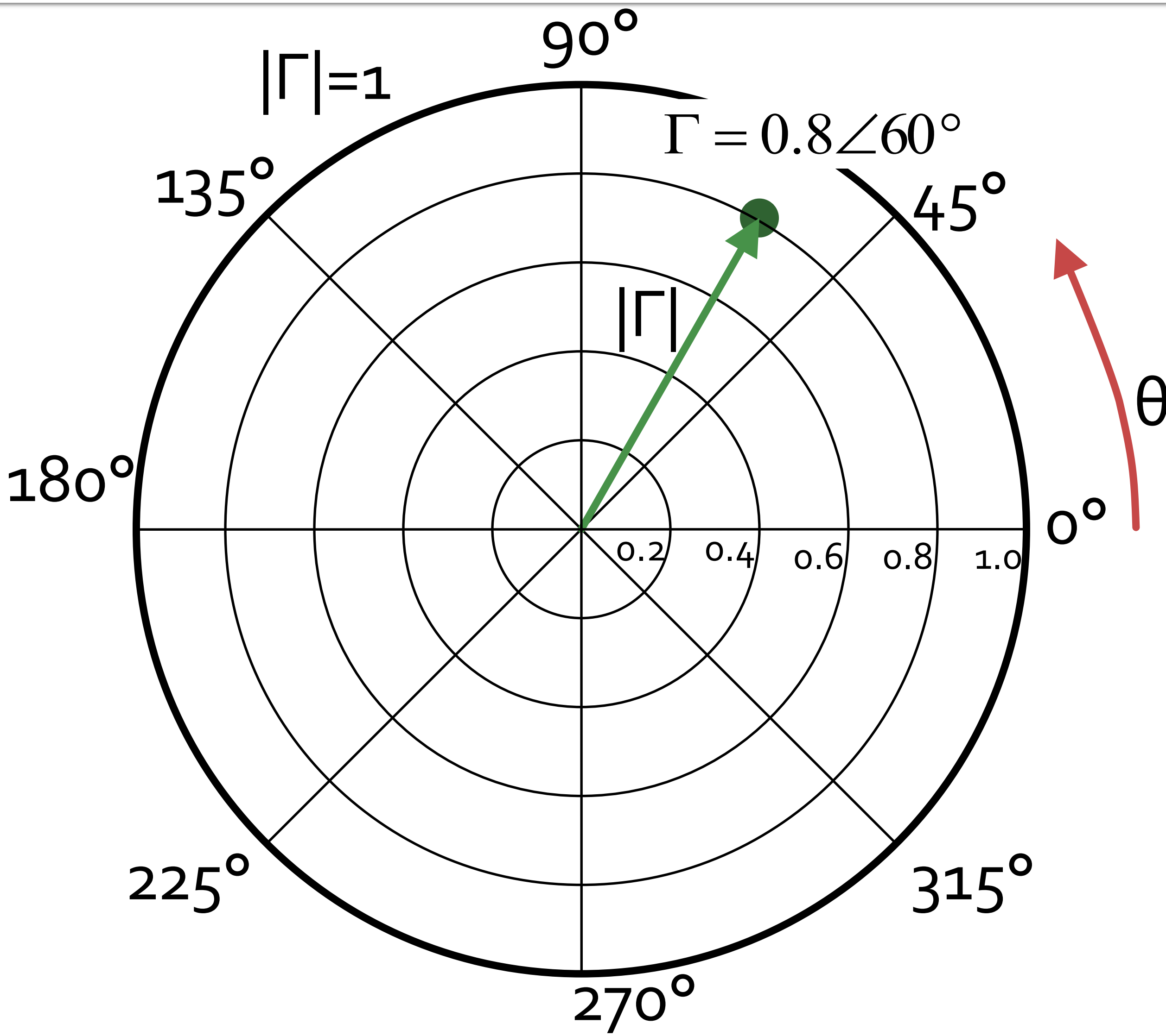
$$\Gamma = 0.8\angle 60°$$

$$\Gamma_r = 0.8 \cdot \cos 60° = 0.4$$

$$\Gamma_i = 0.8 \cdot \sin 60° = 0.693$$

$$\Gamma = 0.8\angle 60° = 0.4 + j \cdot 0.693$$

The Smith Chart, reflection coefficient, Polar coordinate system



$$\Gamma = \Gamma_r + j \cdot \Gamma_i$$

$$\Gamma = |\Gamma| \cdot (\cos\theta + j \cdot \sin\theta)$$

$$\Gamma = |\Gamma| \cdot e^{j\theta}$$

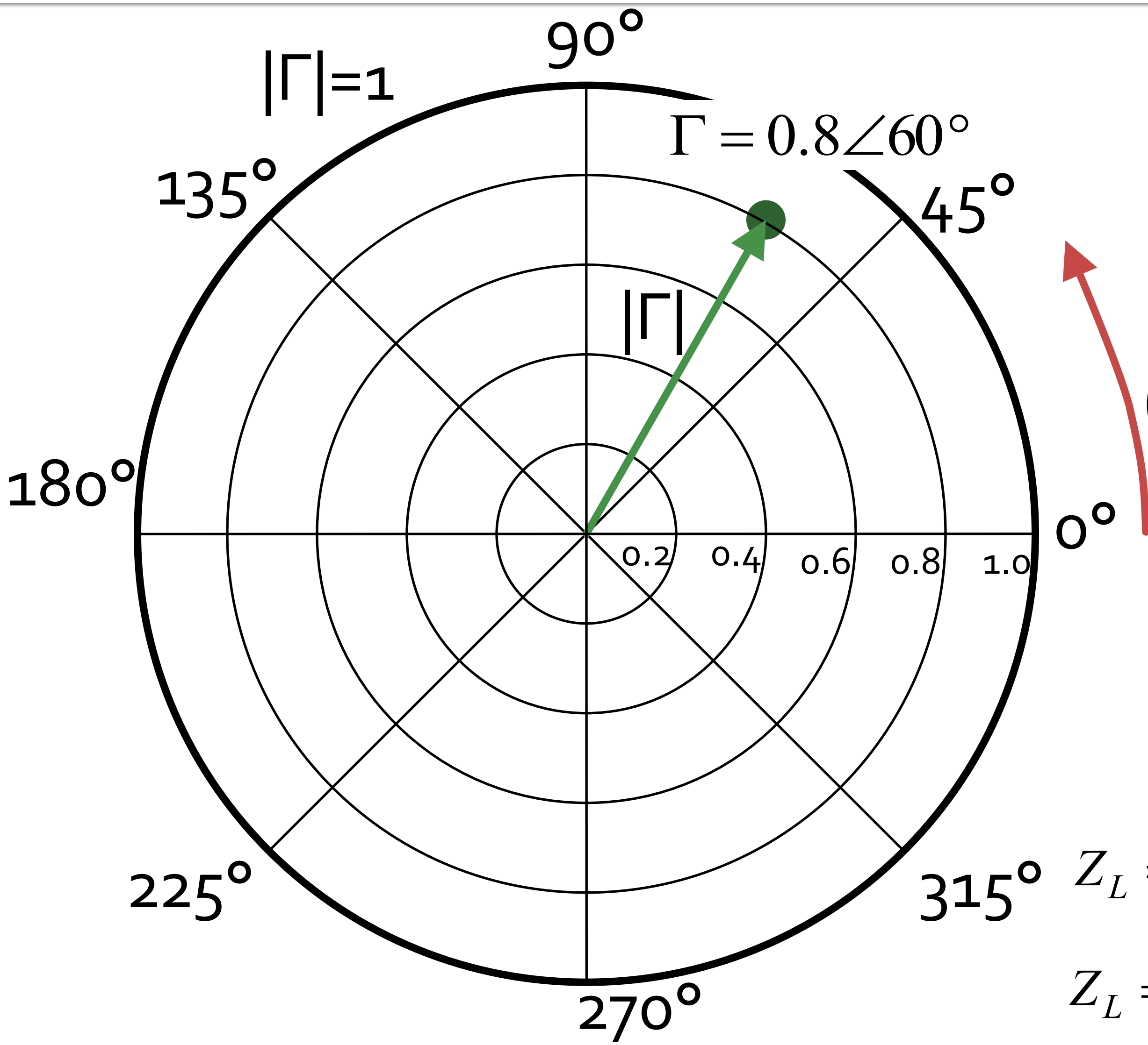
$$\Gamma = |\Gamma| \angle \theta^\circ$$

$$\Gamma = 0.8 \angle 60^\circ$$

$$\Gamma_r = 0.8 \cdot \cos 60^\circ = 0.4$$

$$\Gamma_i = 0.8 \cdot \sin 60^\circ = 0.693$$

The Smith Chart, reflection coefficient, impedance



$$\Gamma = |\Gamma| \cdot e^{j\theta}$$

$$\Gamma = |\Gamma| \angle \theta^{\circ}$$

$$\Gamma = 0.8\angle 60^{\circ}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

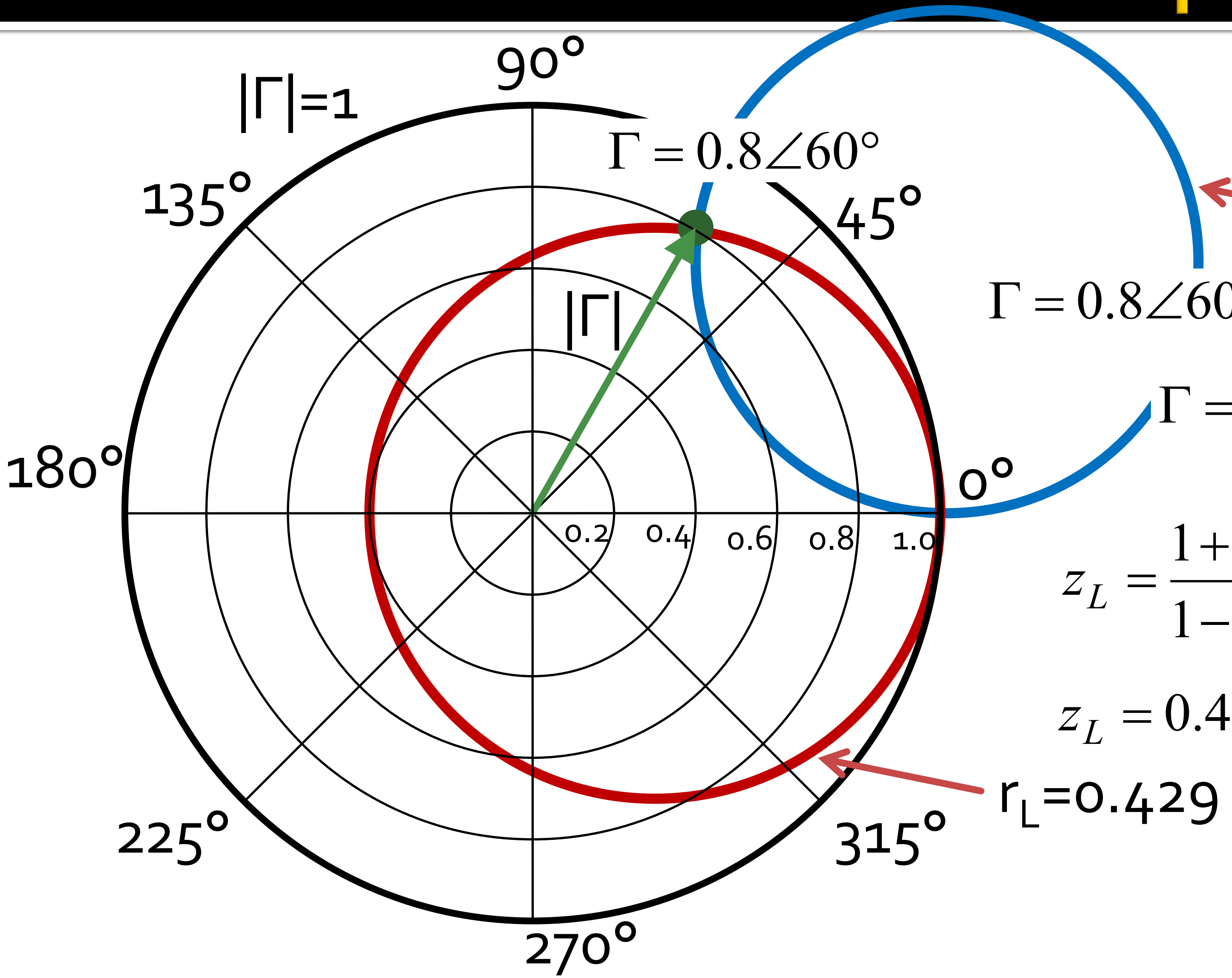
$$z_L = \frac{1+\Gamma}{1-\Gamma} = \frac{1+0.8\angle 60^{\circ}}{1-0.8\angle 60^{\circ}}$$

$$z_L = 0.429 + j \cdot 1.65$$

$$Z_L = Z_0 \cdot \frac{1+\Gamma}{1-\Gamma} = 50\Omega \cdot \frac{1+0.8\angle 60^{\circ}}{1-0.8\angle 60^{\circ}}$$

$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$

Equivalence
reflection coefficient ⇔ impedance



$$\Gamma = 0.8\angle 60^\circ = 0.4 + j\cdot 0.693$$

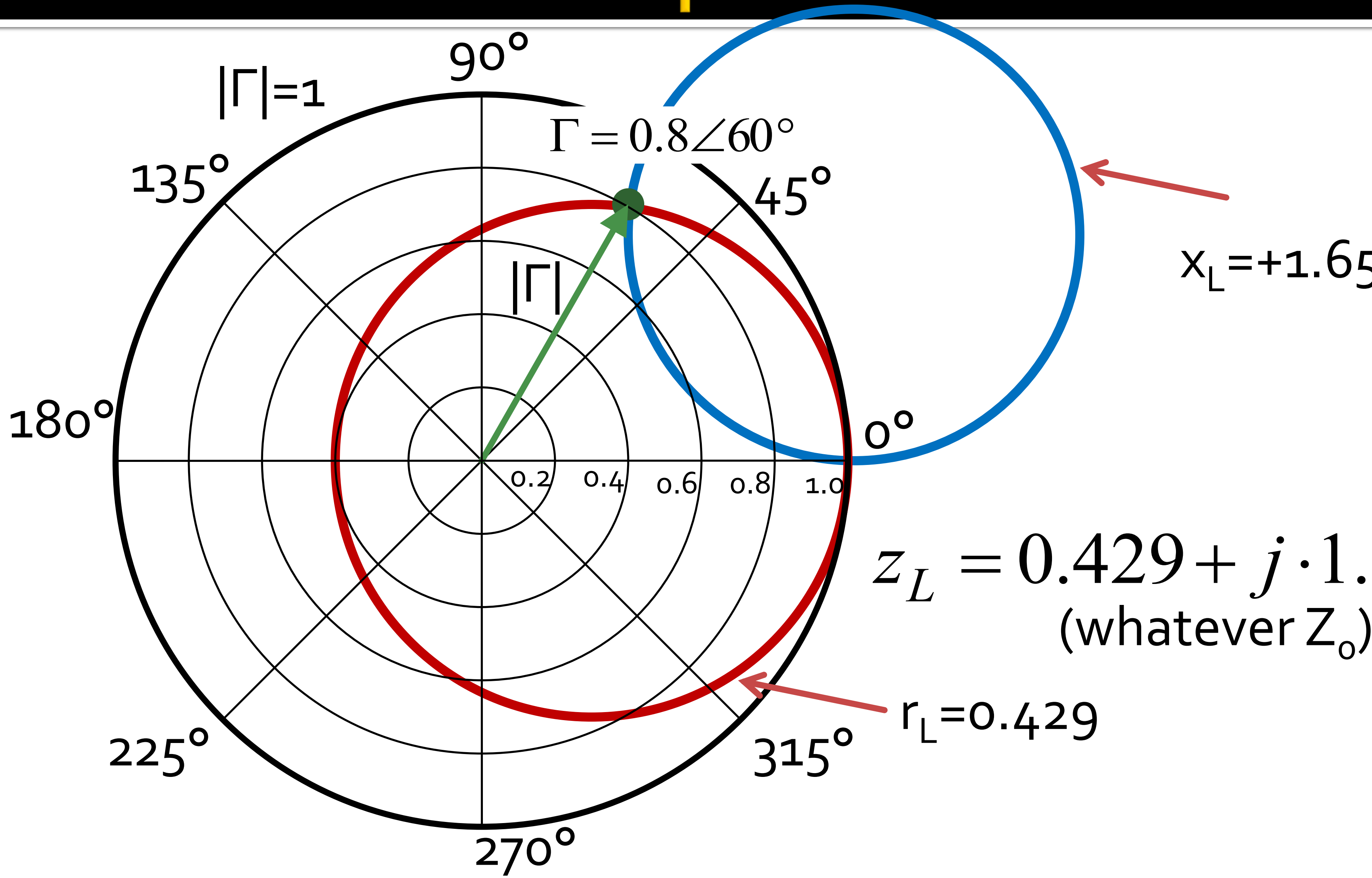
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

$$z_L = \frac{1+\Gamma}{1-\Gamma} = \frac{1+0.8\angle 60^\circ}{1-0.8\angle 60^\circ}$$

$$z_L = 0.429 + j\cdot 1.65$$

(whatever Z_o)

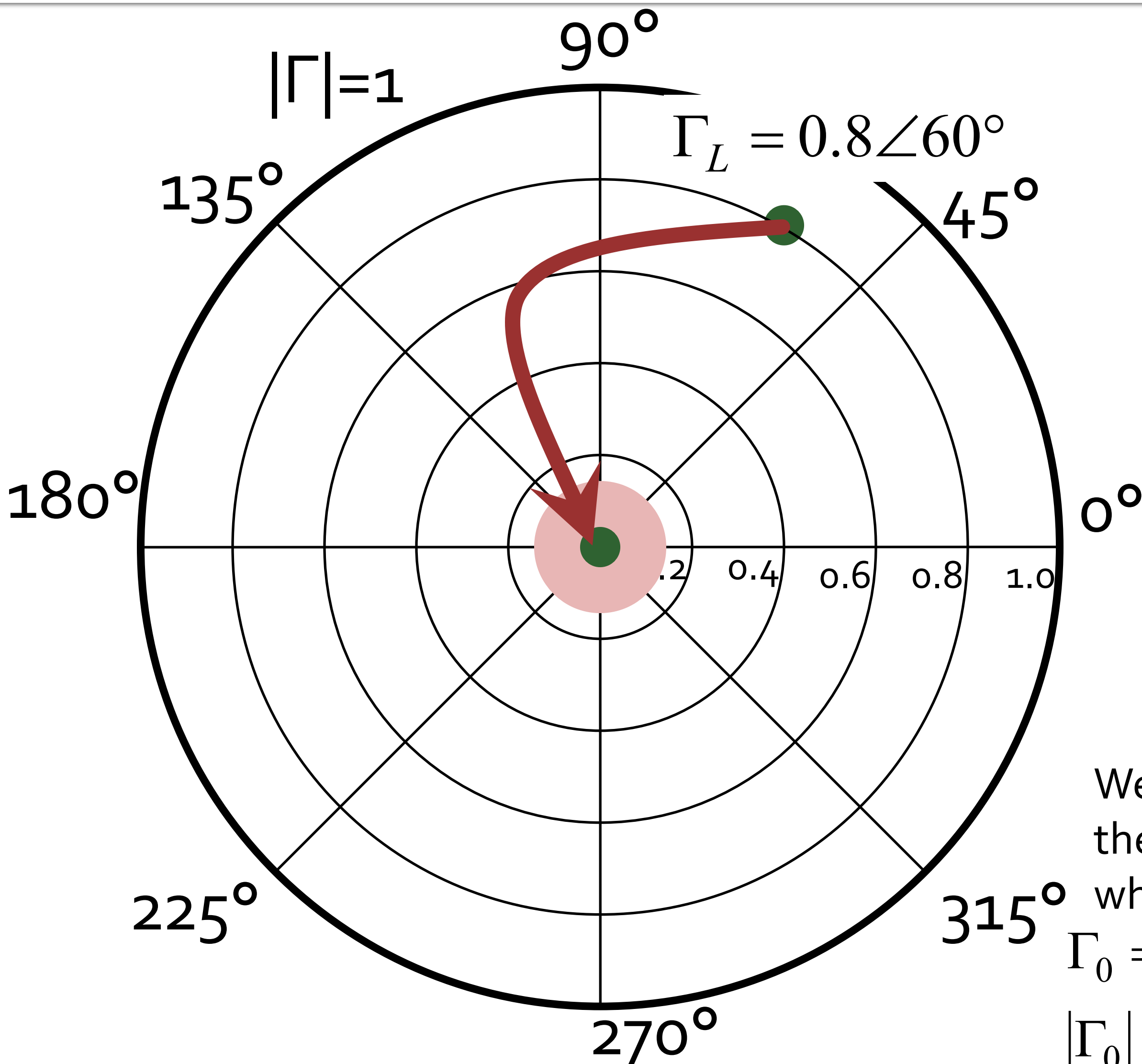
The Smith Chart, reflection coefficient ⇔ impedance

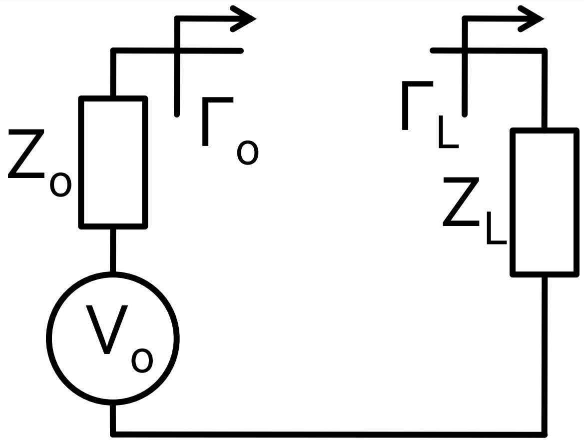


$$z_L = 0.429 + j \cdot 1.65$$

(whatever Z_o)

The Smith Chart, reflection coefficient, matching





Matching Z_L load to Z_o source.
We normalize Z_L over Z_o

$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$

$$z_L = 0.429 + j \cdot 1.65$$

$$\Gamma_L = 0.8\angle 60°$$

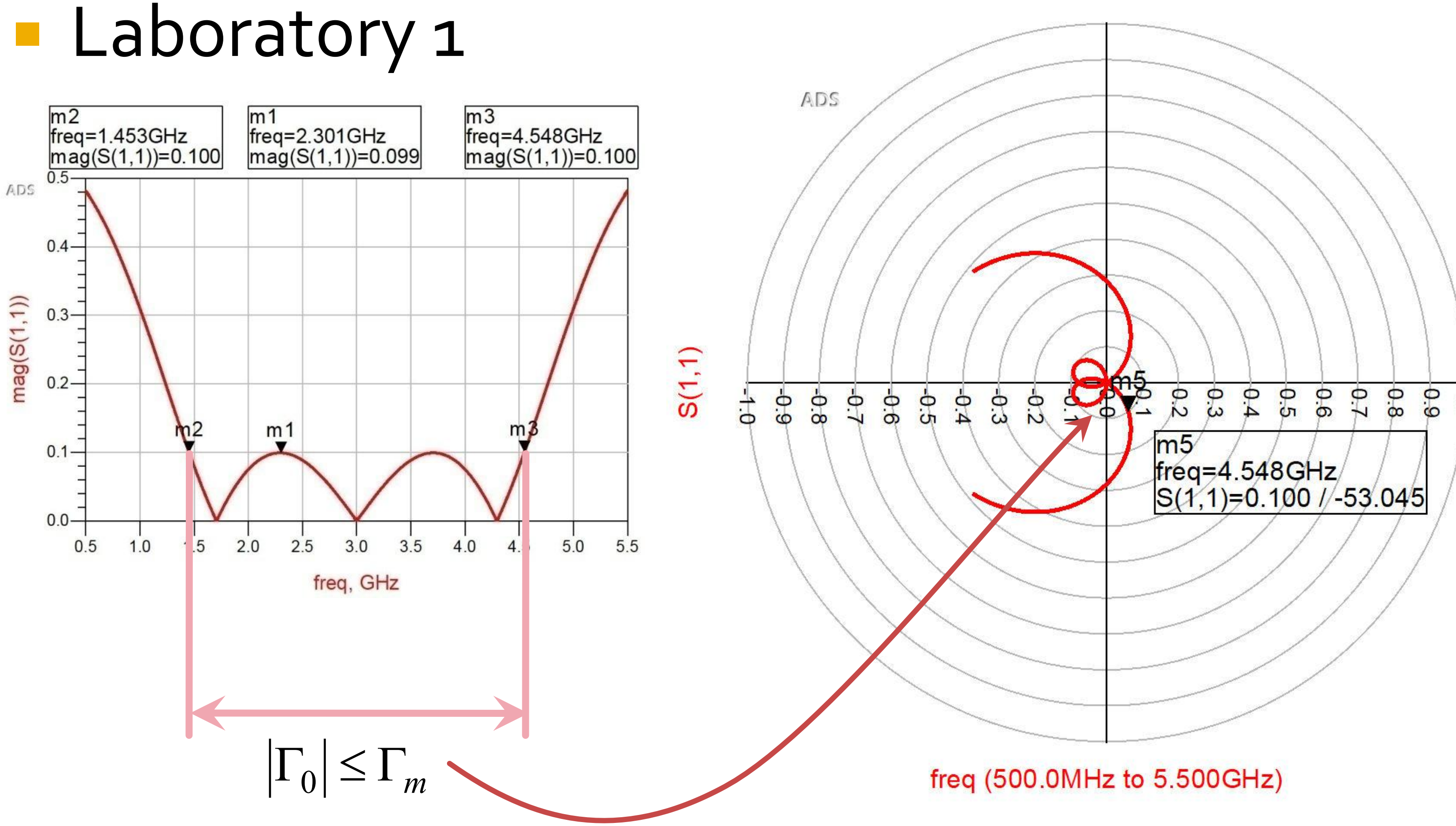
We must move the point denoting the reflection coefficient in the area where with a Z_o source we have:

$\Gamma_0 = 0$ perfect match

$|\Gamma_0| \le \Gamma_m$ “good enough” match

Example

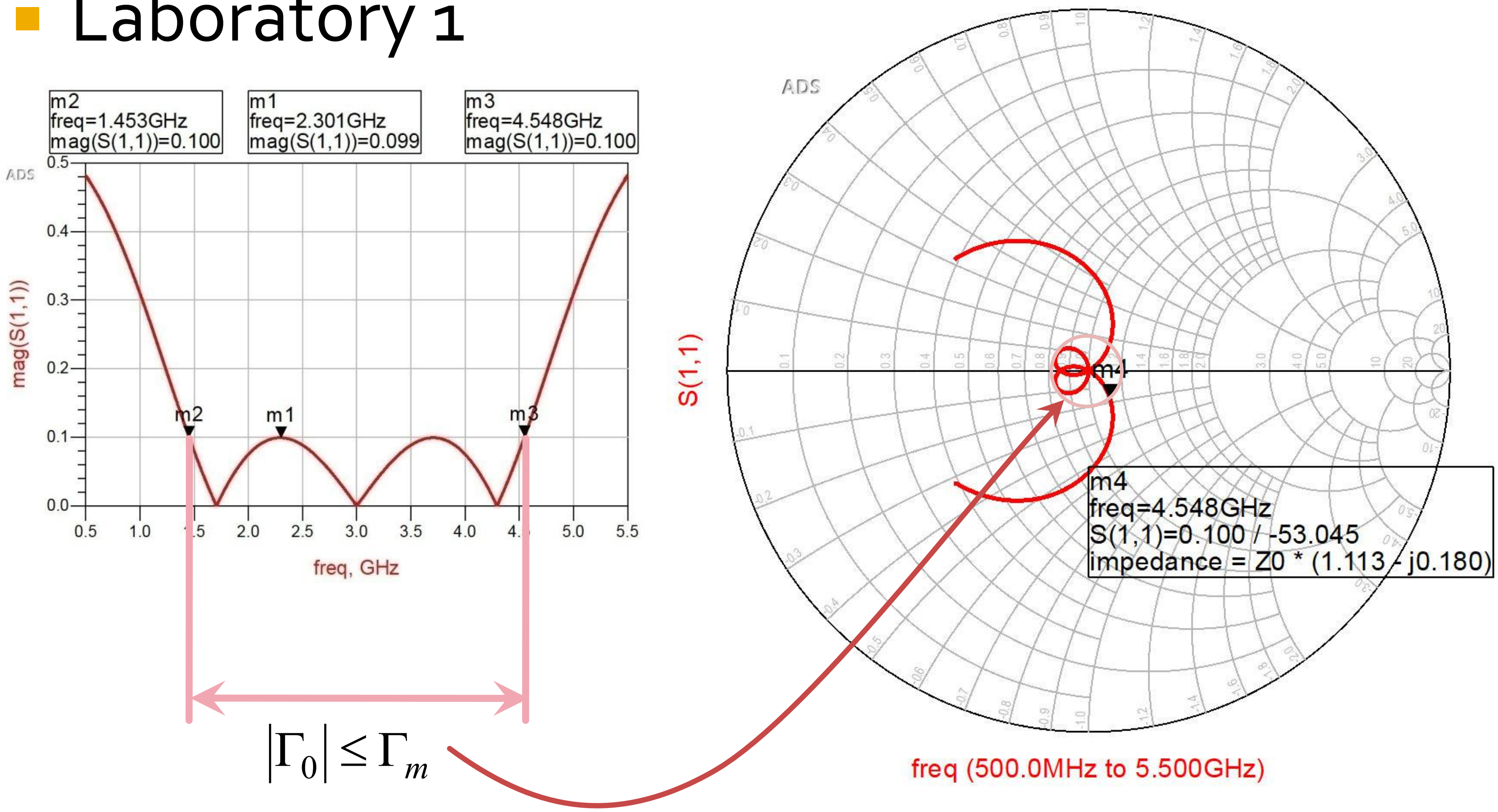
- Laboratory 1



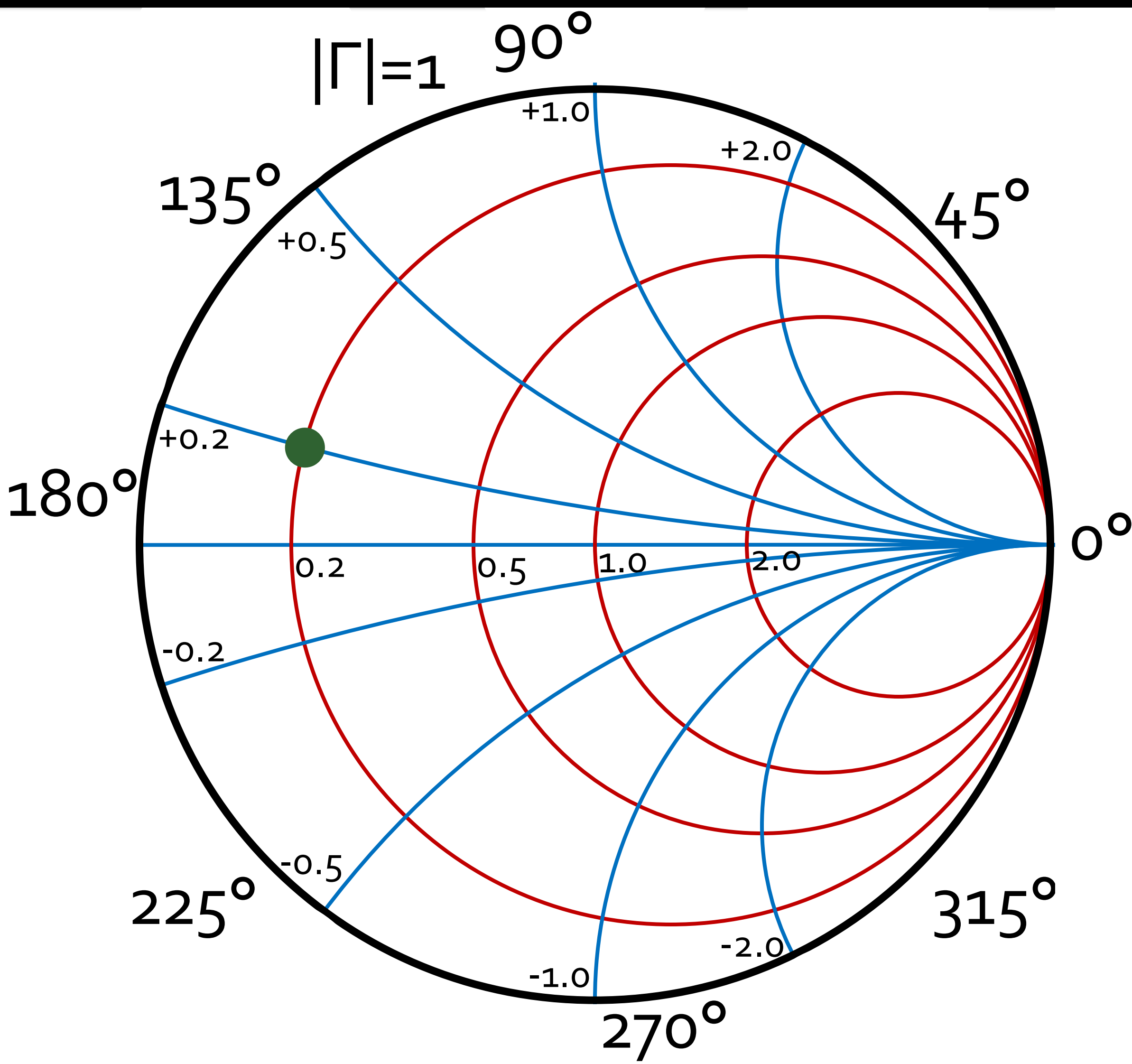
$$|\Gamma_0| \le \Gamma_m$$

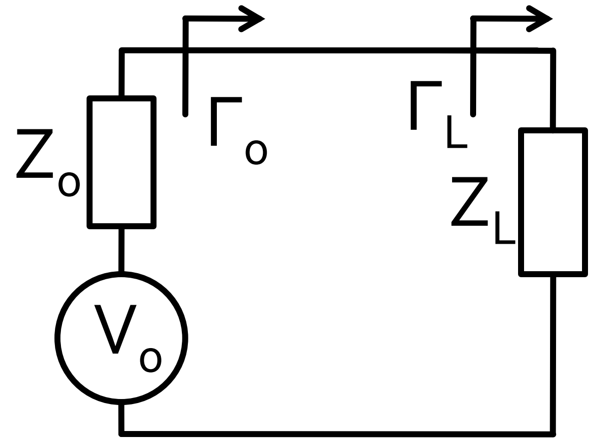
Example

- Laboratory 1



The Smith Chart, impedance/reflection coefficient





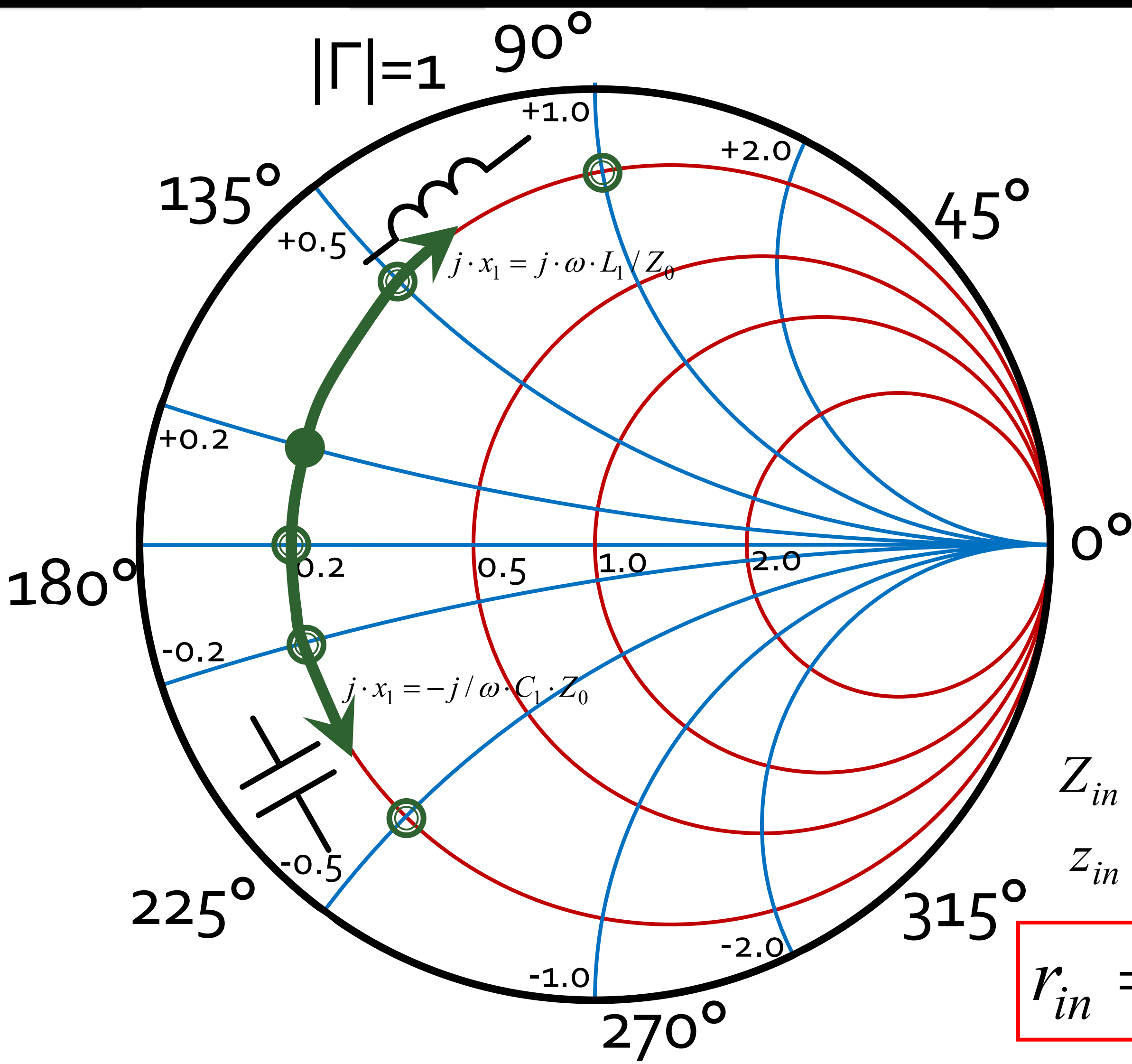
$$Z_0 = 50\Omega$$

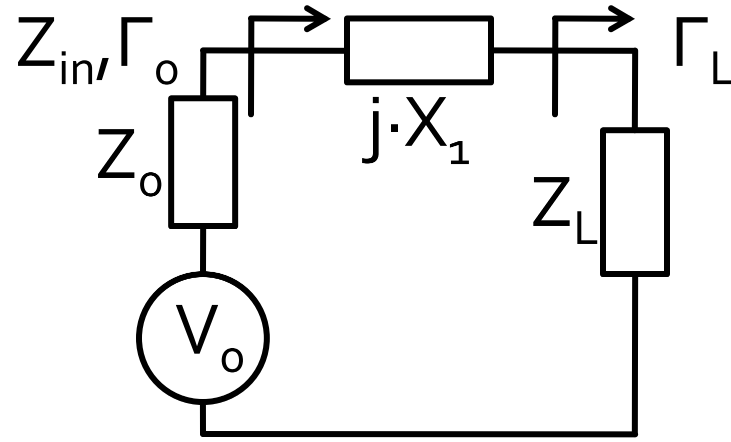
$$Z_L = 10\Omega + j \cdot 10\Omega$$

$$z_L = 0.2 + j \cdot 0.2$$

$$\Gamma_L = \Gamma_0 = 0.678\angle 156.5$$

The Smith Chart, series reactance





$$Z_0 = 50\Omega$$

$$Z_L = R_L + j \cdot X_L = 10\Omega + j \cdot 10$$

$$z_L = r_L + j \cdot x_L = 0.2 + j \cdot 0.2$$

$$\Gamma_L = 0.678\angle 156.5°$$

$$Z_{in} = Z_L + j \cdot X_1 = R_L + j \cdot (X_L + X_1$$

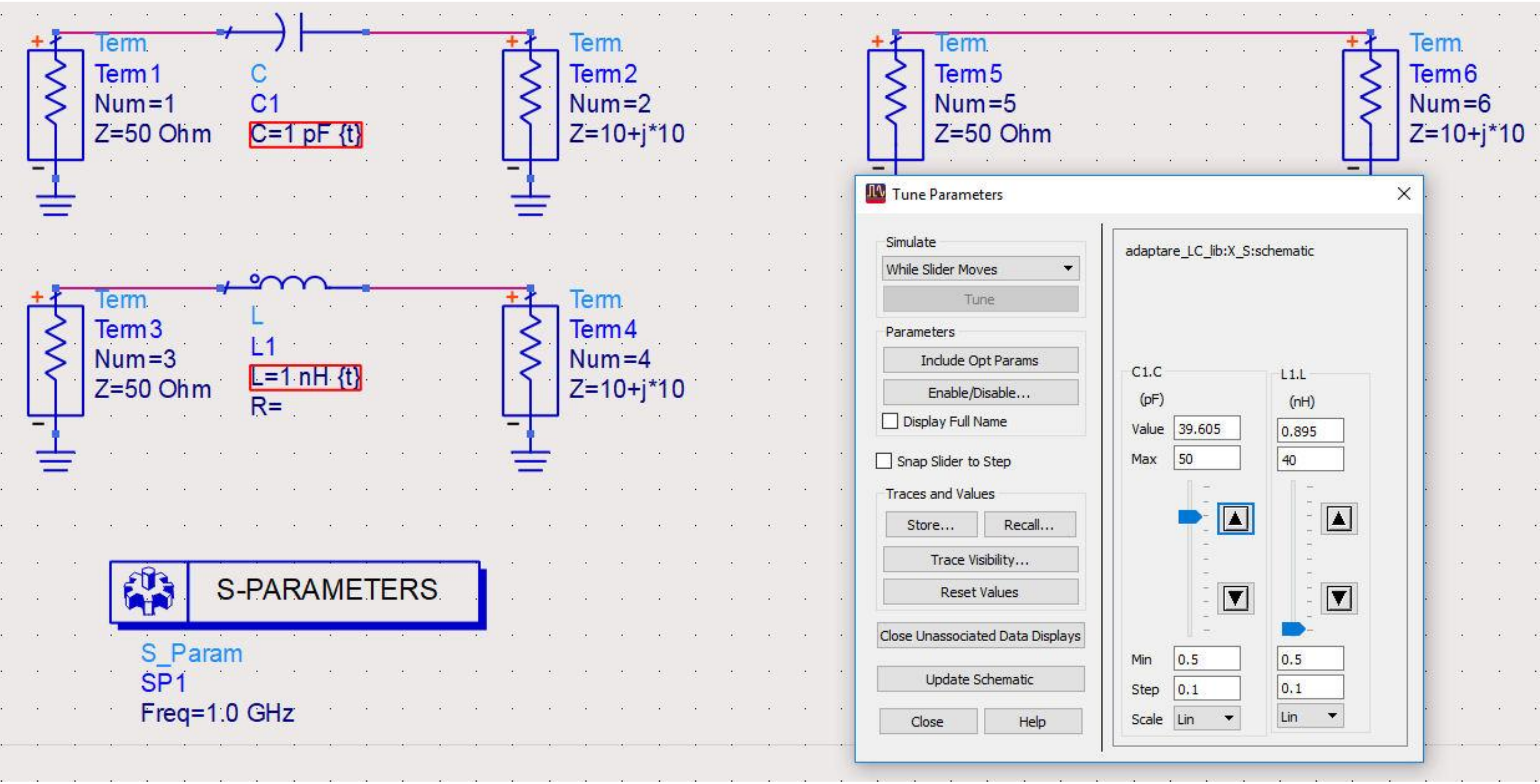
$$z_{in} = r_L + j \cdot (x_L + x_1)$$

$$r_{in} = r_L$$

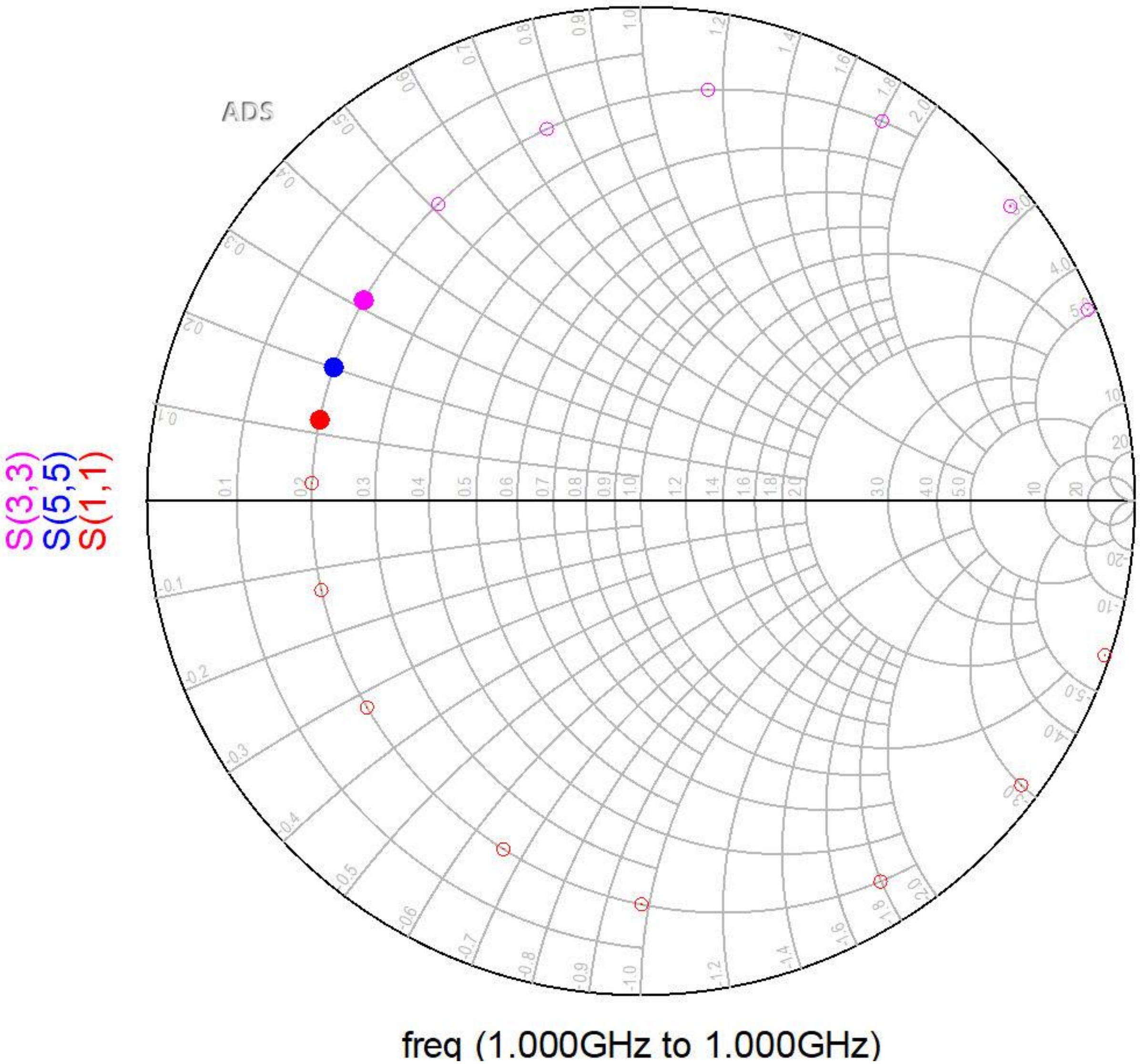
$$j \cdot x_1 = j \cdot \omega \cdot L_1 / Z_0 > 0$$

$$j \cdot x_1 = -j / \omega \cdot C_1 \cdot Z_0 < 0$$

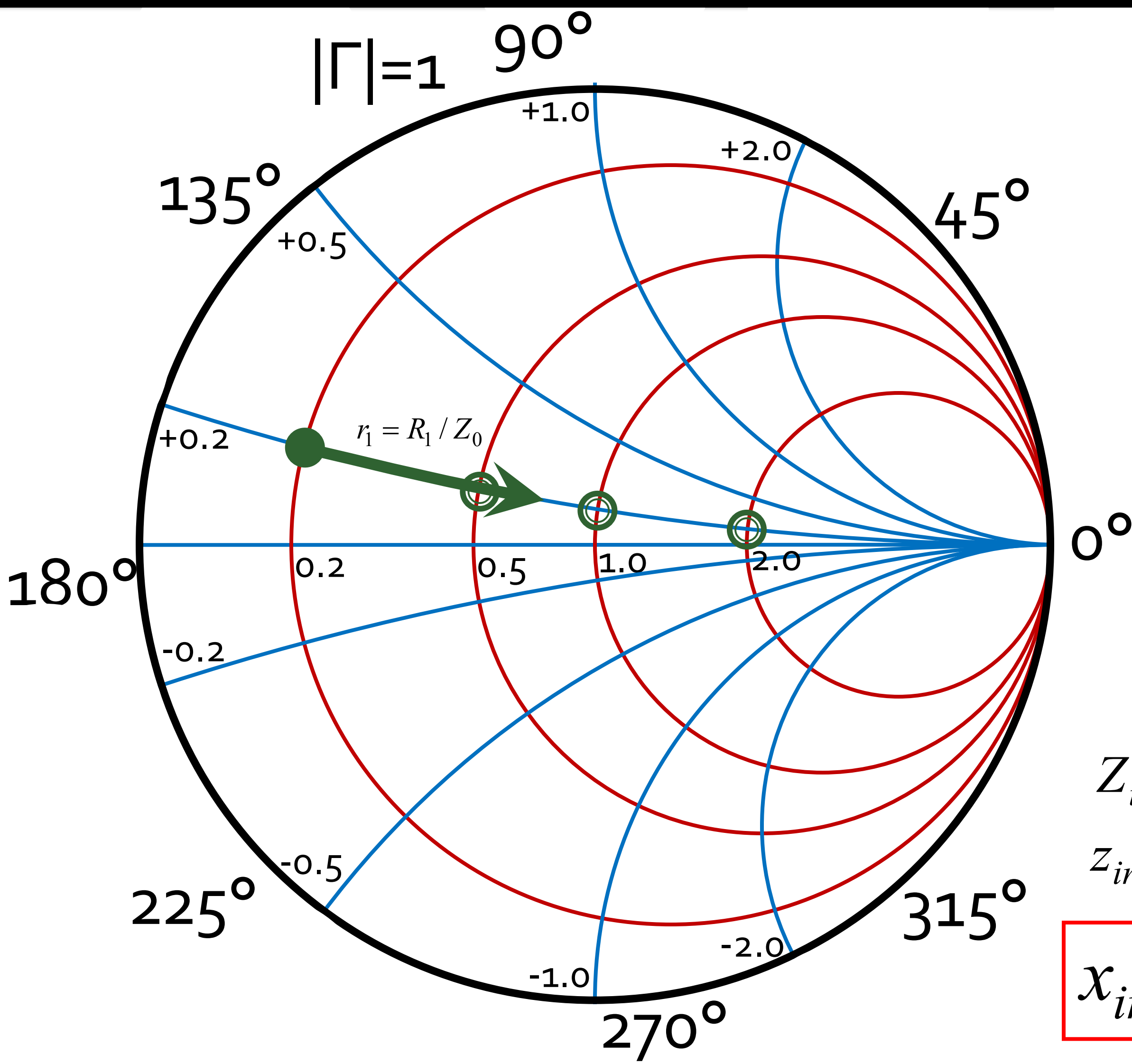
ADS, Smith Chart, series reactance

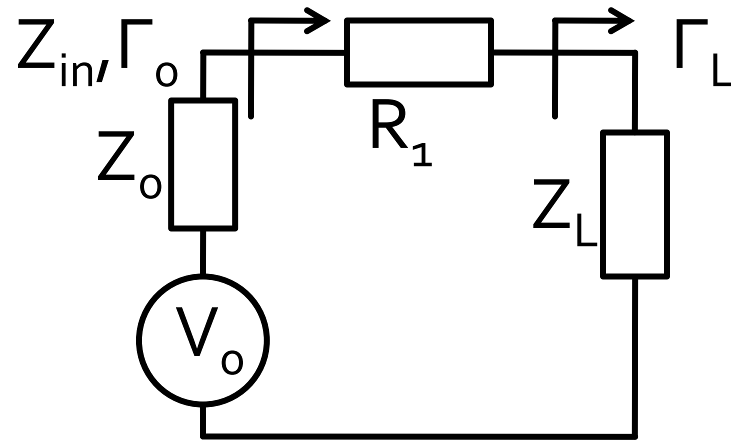


ADS, Smith Chart, series reactance



The Smith Chart, series resistance





$$Z_0 = 50\Omega$$

$$Z_L = R_L + j \cdot X_L = 10\Omega + j \cdot 10$$

$$z_L = r_L + j \cdot x_L = 0.2 + j \cdot 0.2$$

$$\Gamma_L = 0.678\angle 156.5°$$

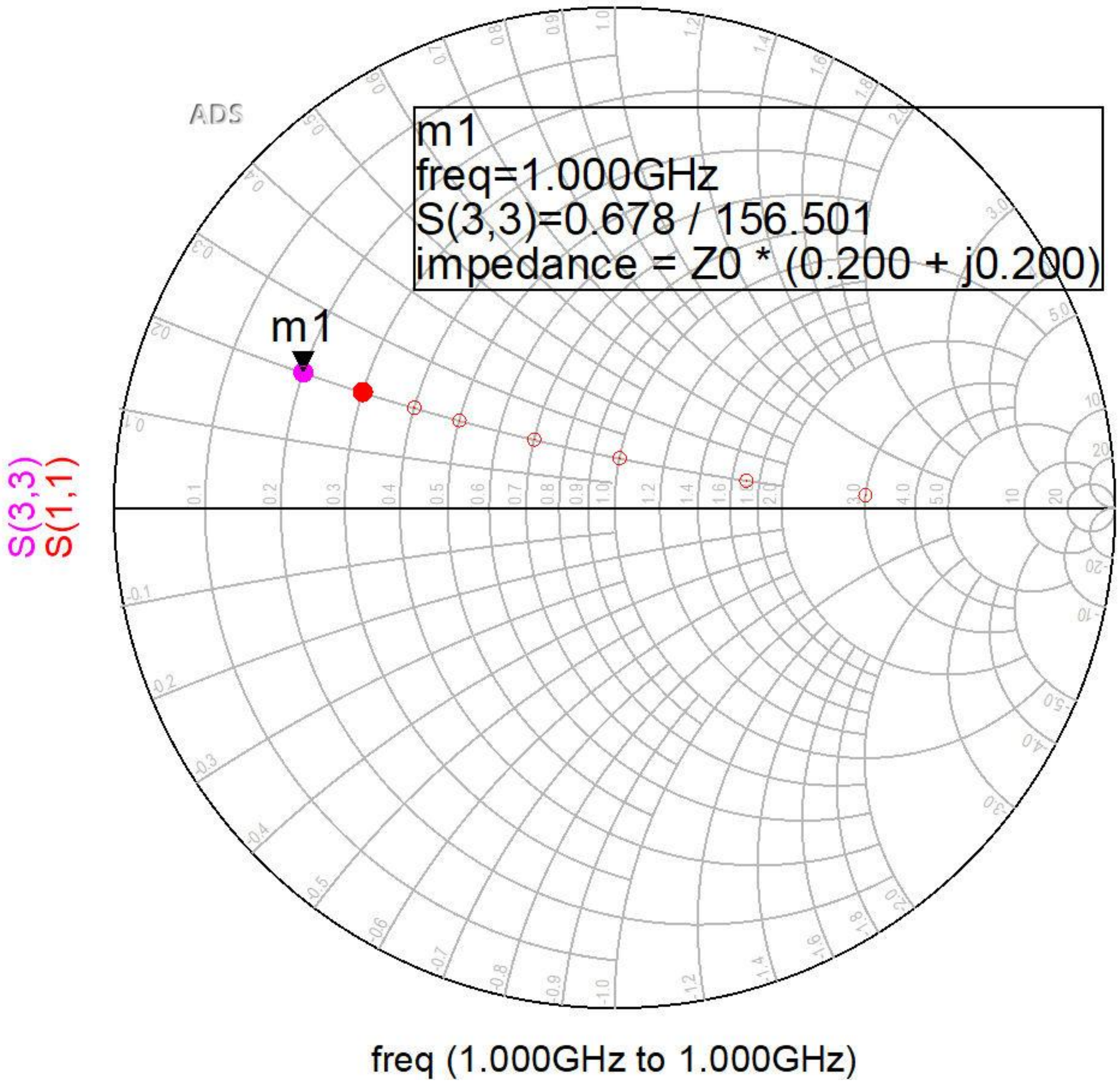
$$Z_{in} = Z_L + R_1 = \left(R_L + R_1\right) + j \cdot X_L$$

$$z_{in} = z_L + r_1 = \left(r_L + r_1\right) + j \cdot x_L$$

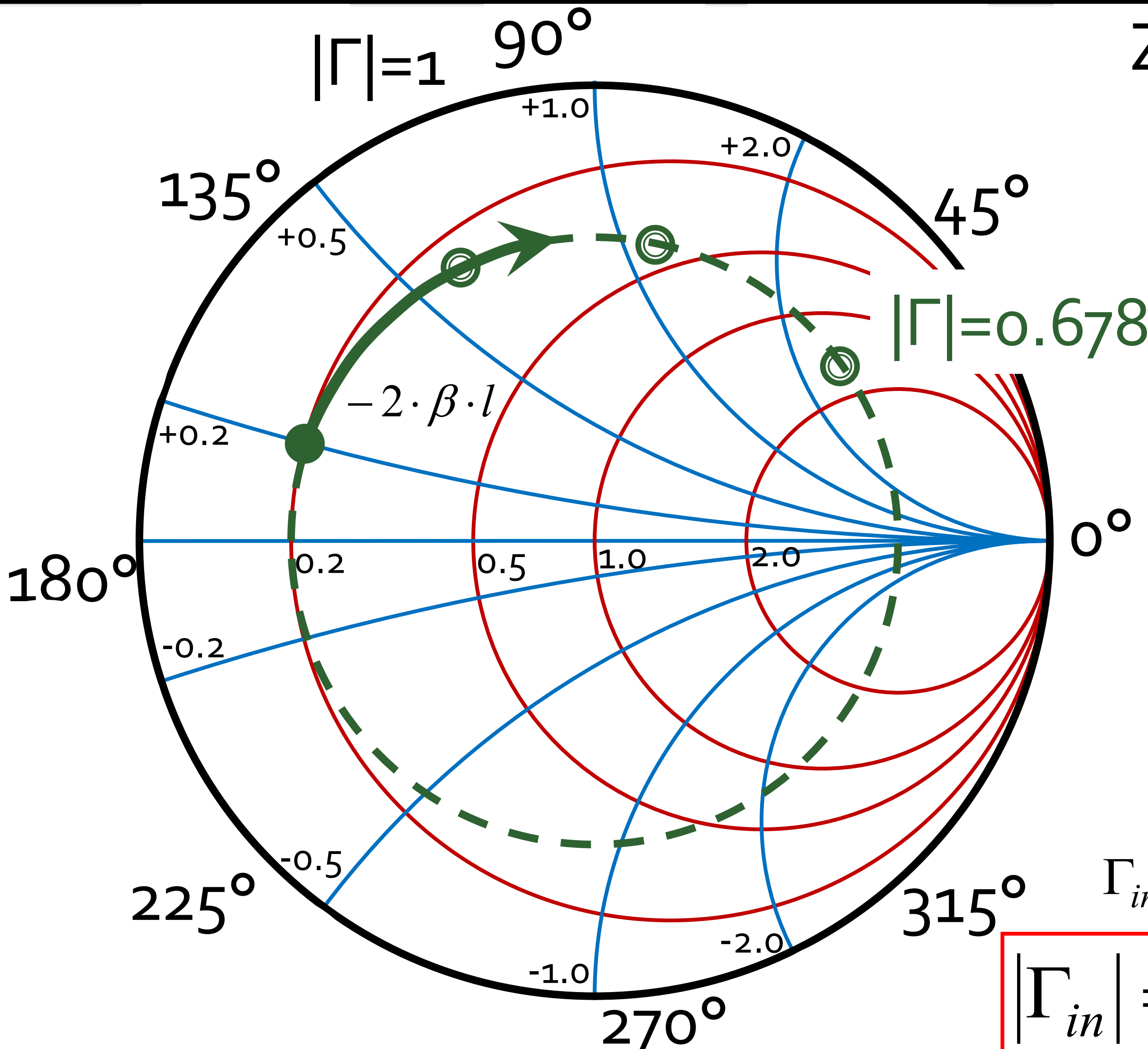
$$x_{in} = x_L$$

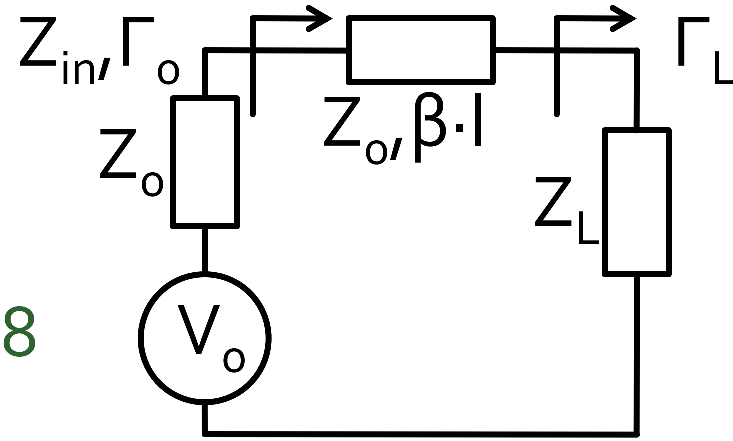
$$r_{in} = r_L + R_1 / Z_0$$

ADS, Smith Chart, series resistance



The Smith Chart, series transmission line, Z_o





$$Z_0 = 50\Omega$$

$$Z_L = R_L + j\cdot X_L = 10\Omega + j\cdot 10$$

$$z_L = r_L + j\cdot x_L = 0.2 + j\cdot 0.2$$

$$\Gamma_L = 0.678\angle 156.5°$$

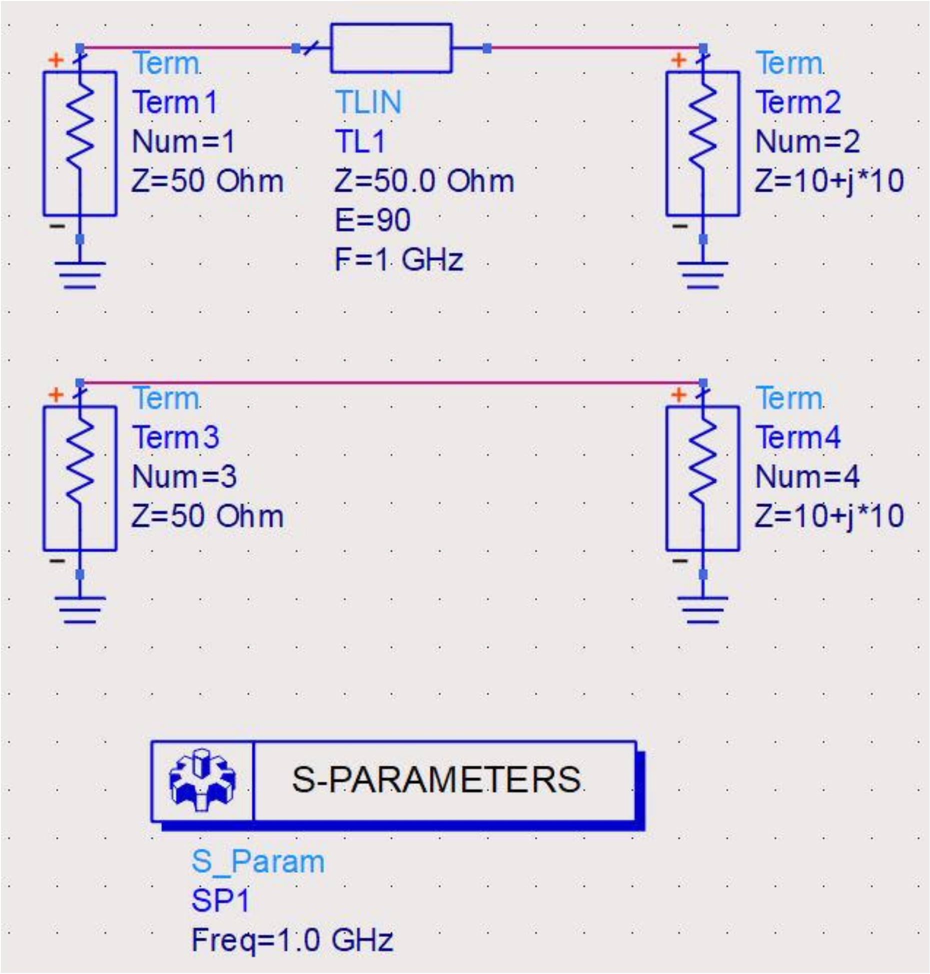
$$Z_{in} = Z_0 \cdot \frac{1+\Gamma_L \cdot e^{-2j\cdot\beta\cdot l}}{1-\Gamma_L \cdot e^{-2j\cdot\beta\cdot l}}$$

$$\Gamma_{in} = \Gamma_L \cdot e^{-2j\cdot\beta\cdot l}$$

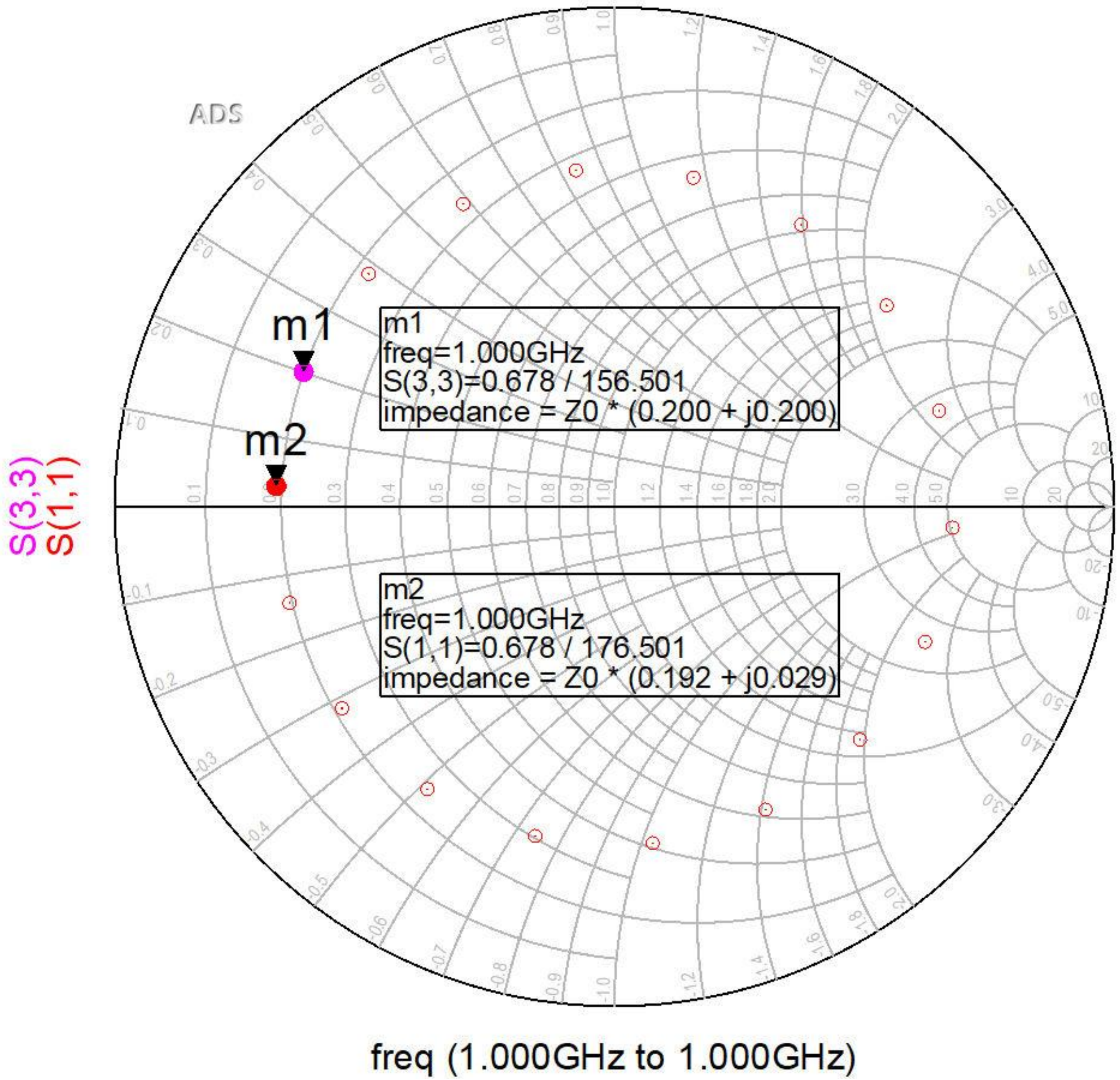
$$\left|\Gamma_{in}\right| = \left|\Gamma_L\right|$$

$$\arg(\Gamma_{in}) = \arg(\Gamma_L) - 2\cdot\beta\cdot$$

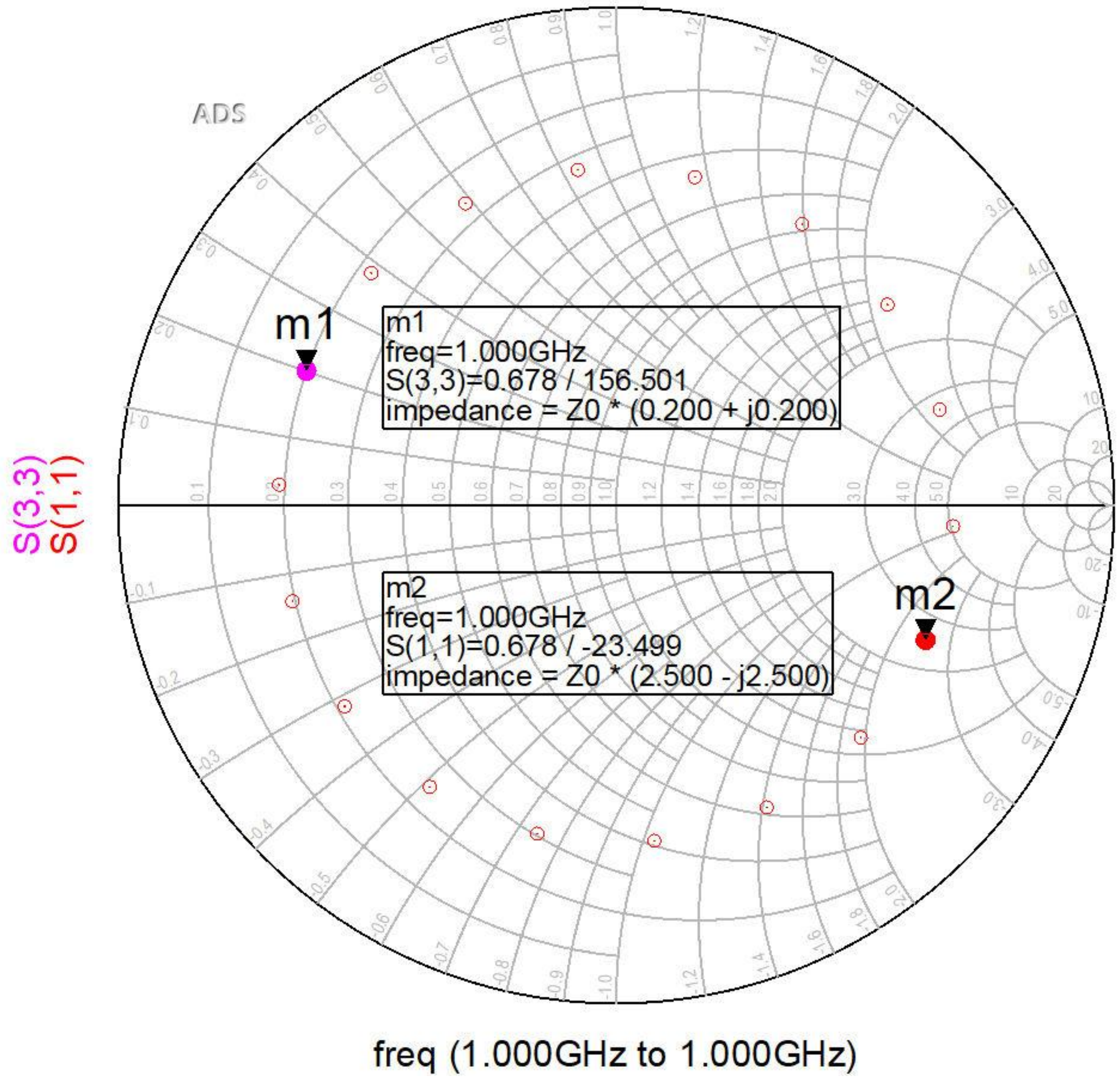
ADS, Smith Chart, series transmission line



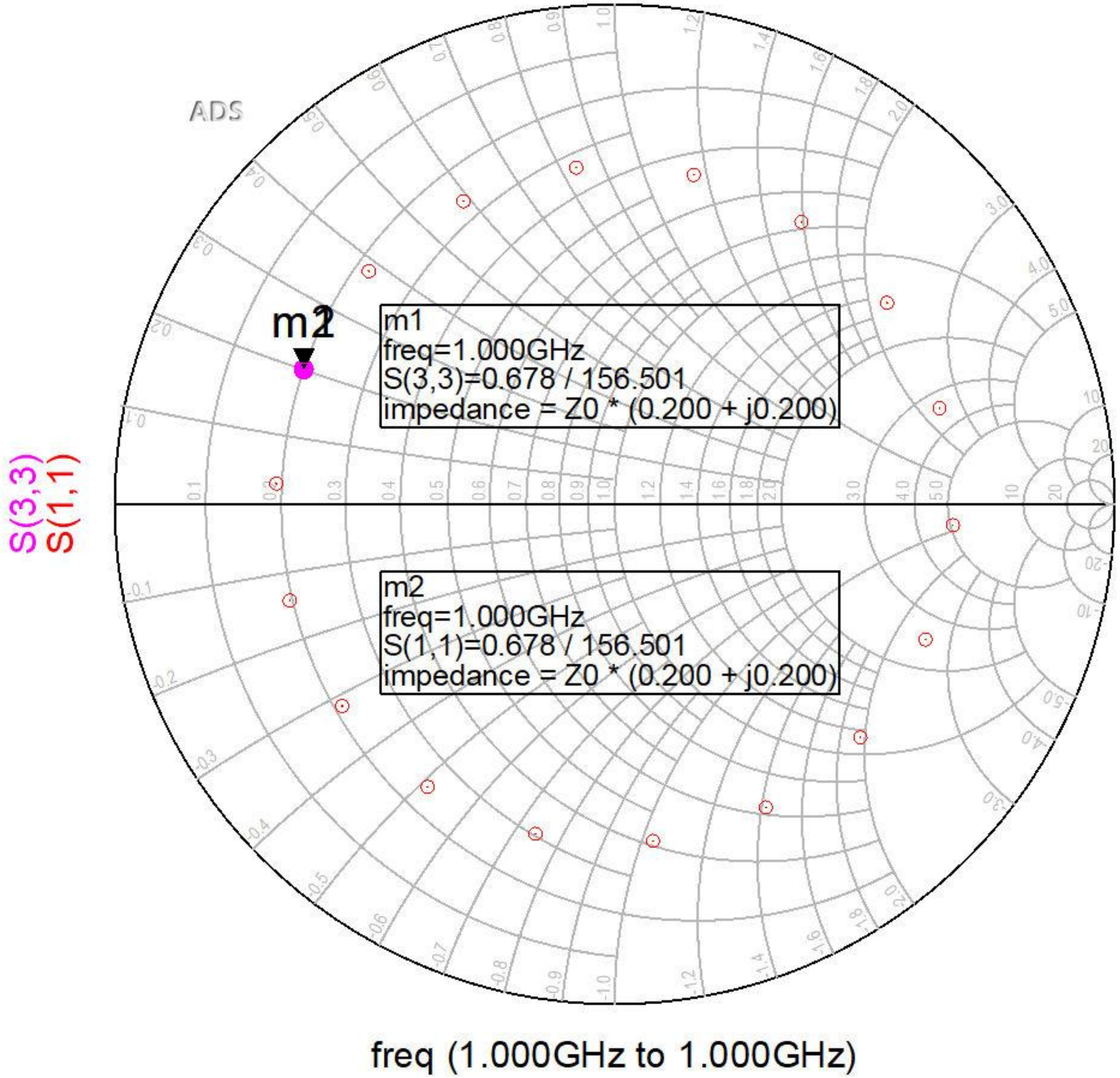
ADS, Smith Chart, series transmission line



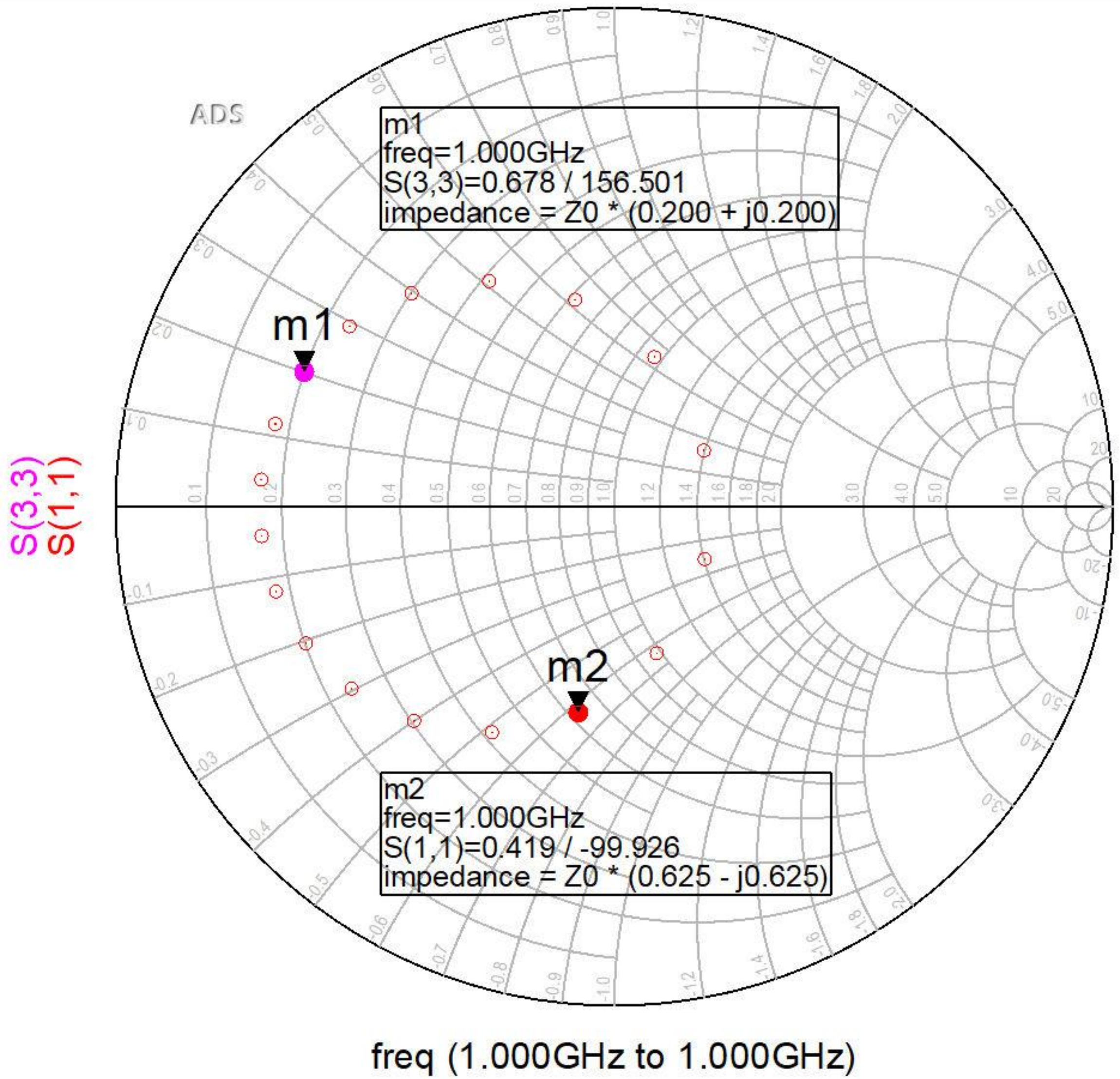
ADS, Smith Chart, series transmission line, E=90°



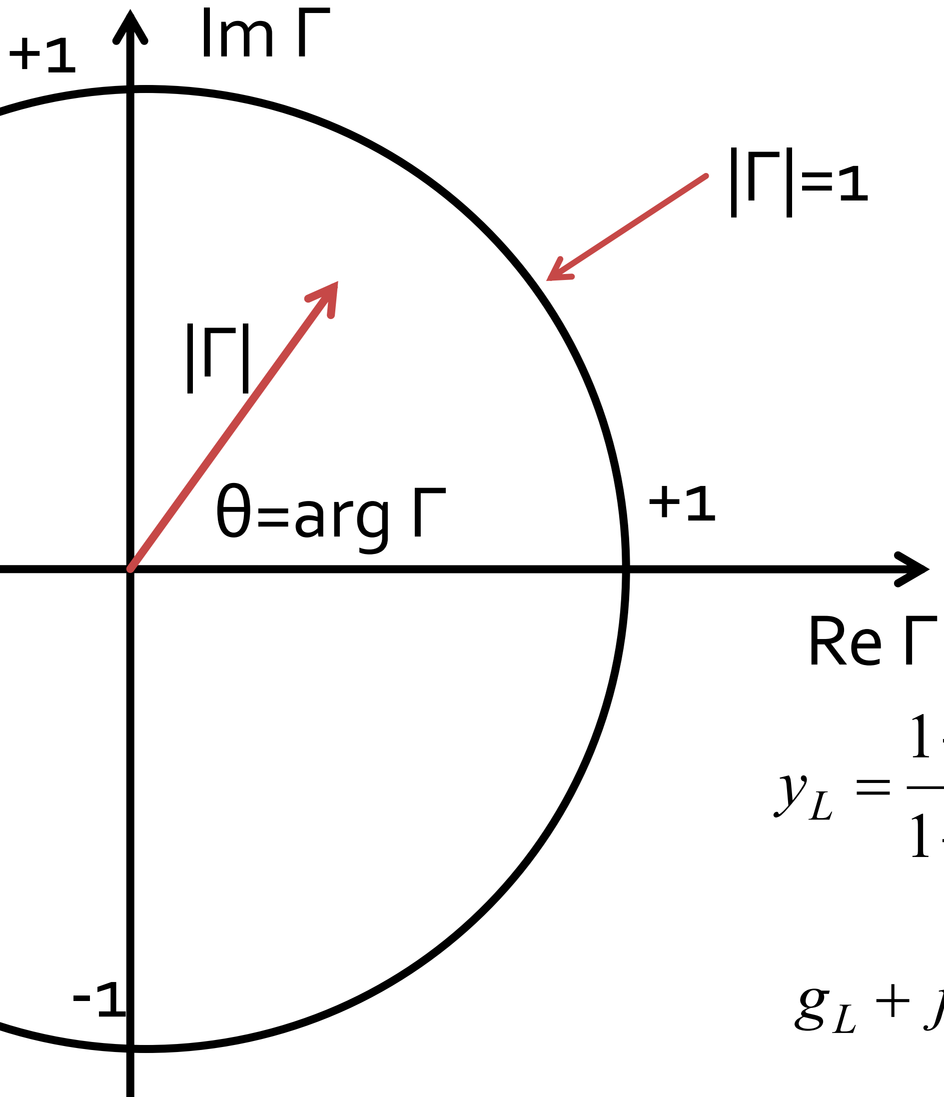
ADS, Smith Chart, series transmission line, E=180°



ADS, Smith Chart, series transmission line, Z=25Ω≠Z_o



The Admittance Smith Chart



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1} = |\Gamma| \cdot e^{j\theta}$$

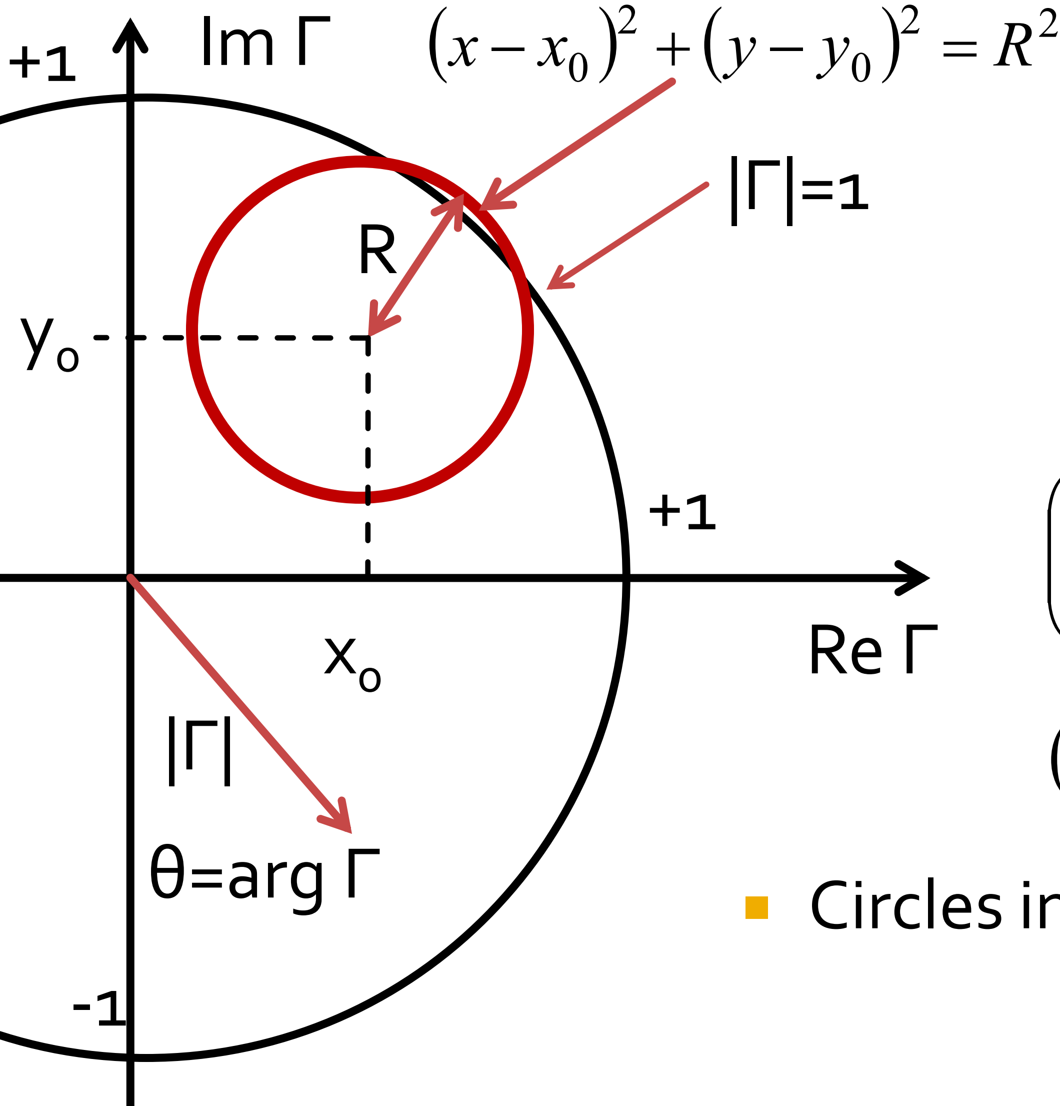
$$\Gamma = \Gamma_r + j \cdot \Gamma_i$$

$$z_L = \frac{1 + |\Gamma| \cdot e^{j\theta}}{1 - |\Gamma| \cdot e^{j\theta}} = r_L + j \cdot x_L$$

$$y_L = \frac{1 - |\Gamma| \cdot e^{j\theta}}{1 + |\Gamma| \cdot e^{j\theta}} = \frac{1}{r_L + j \cdot x_L} = g_L + j \cdot b_L$$

$$g_L + j \cdot b_L = \frac{(1 - \Gamma_r) - j \cdot \Gamma_i}{(1 + \Gamma_r) + j \cdot \Gamma_i}$$

The Admittance Smith Chart



$$g_L = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1+\Gamma_r)^2+\Gamma_i^2}$$

$$b_L = \frac{-2\cdot\Gamma_i}{(1+\Gamma_r)^2+\Gamma_i^2}$$

- Rearranged

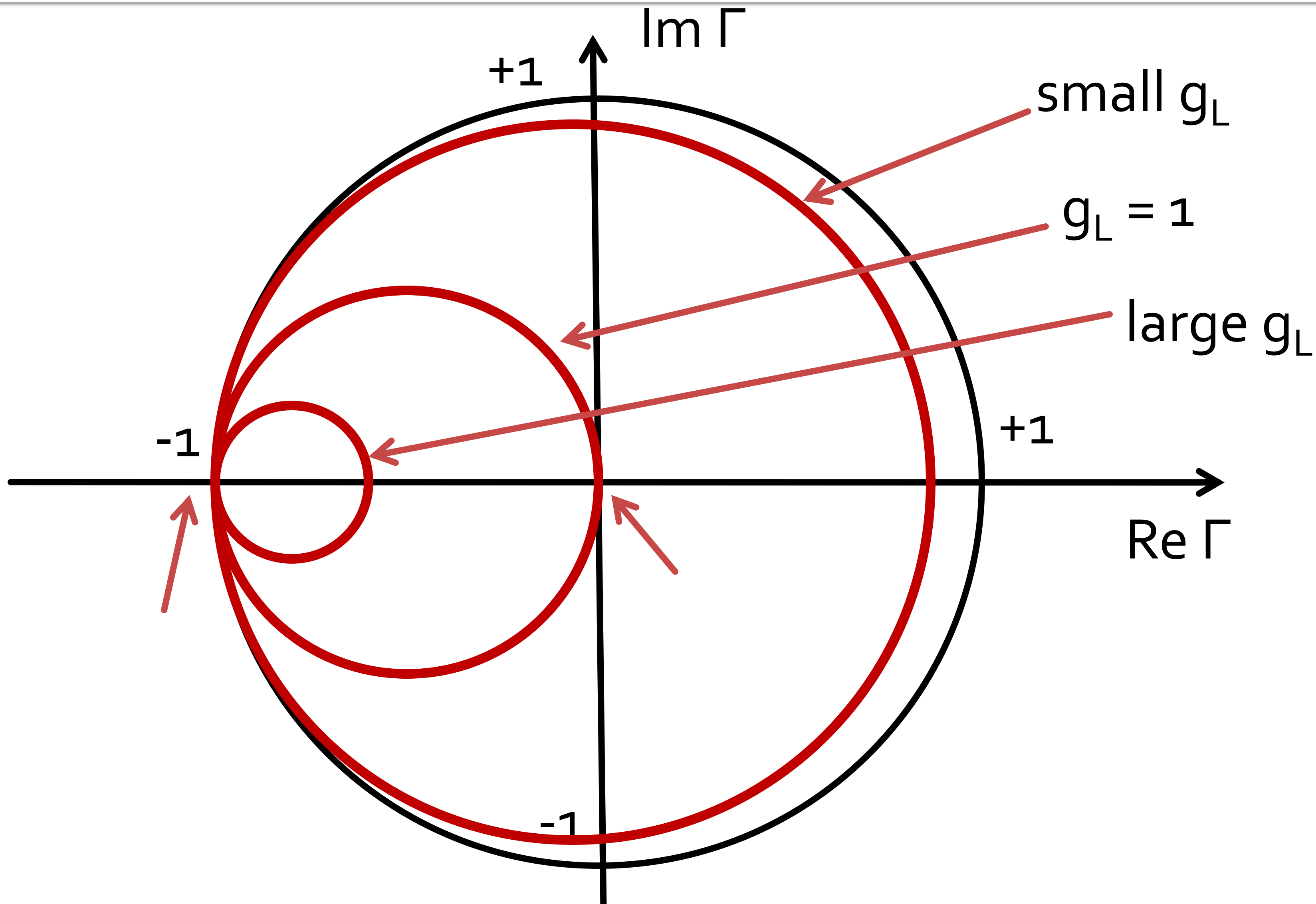
$$\left(\Gamma_r+\frac{g_L}{1+g_L}\right)^2+\Gamma_i^2=\left(\frac{1}{1+g_L}\right)^2$$

$$(\Gamma_r+1)^2+\left(\Gamma_i+\frac{1}{b_L}\right)^2=\left(\frac{1}{b_L}\right)^2$$

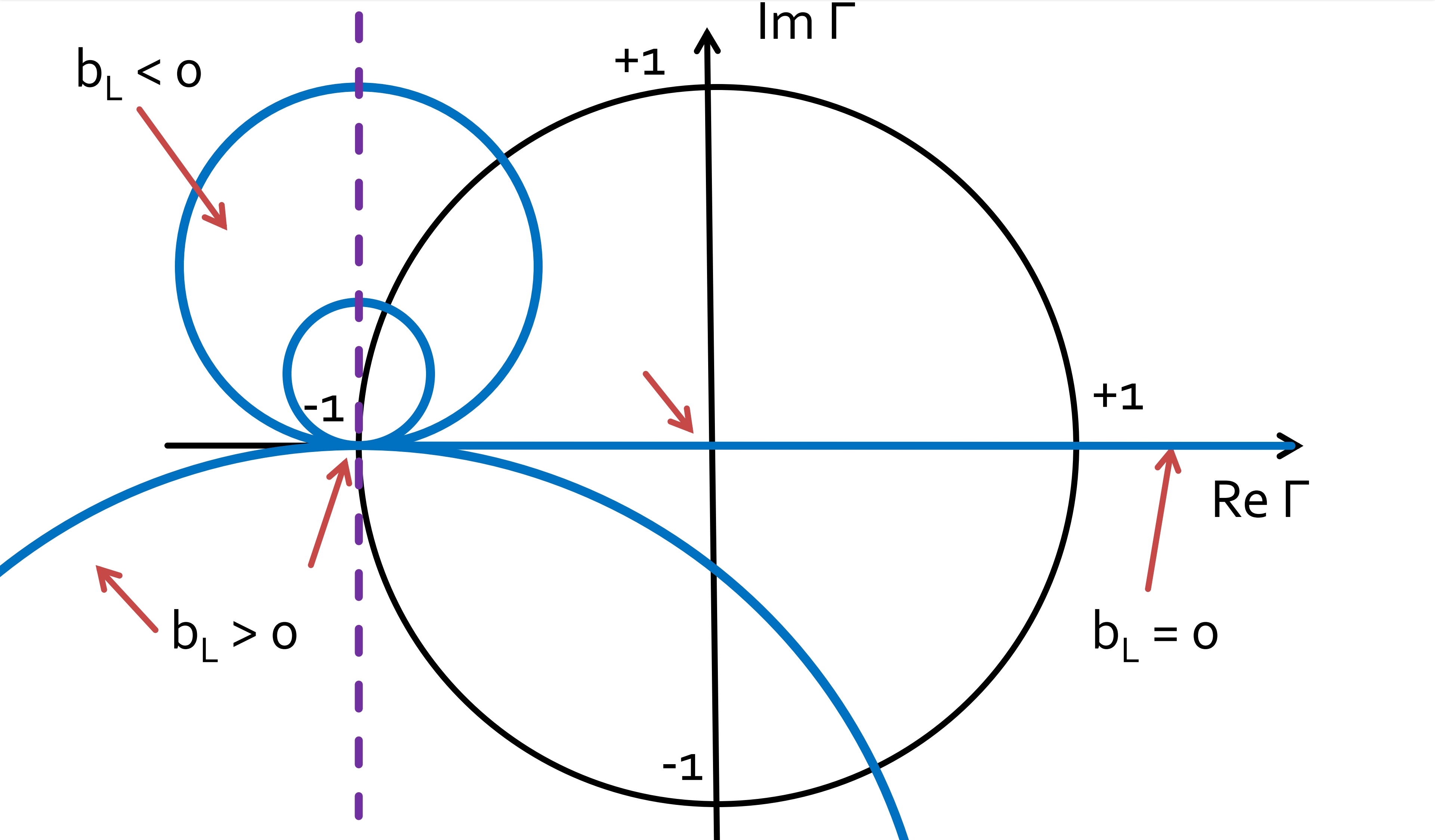
- Circles in the (Γ_r , Γ_i) complex plane

$$(x-x_0)^2+(y-y_0)^2=R^2$$

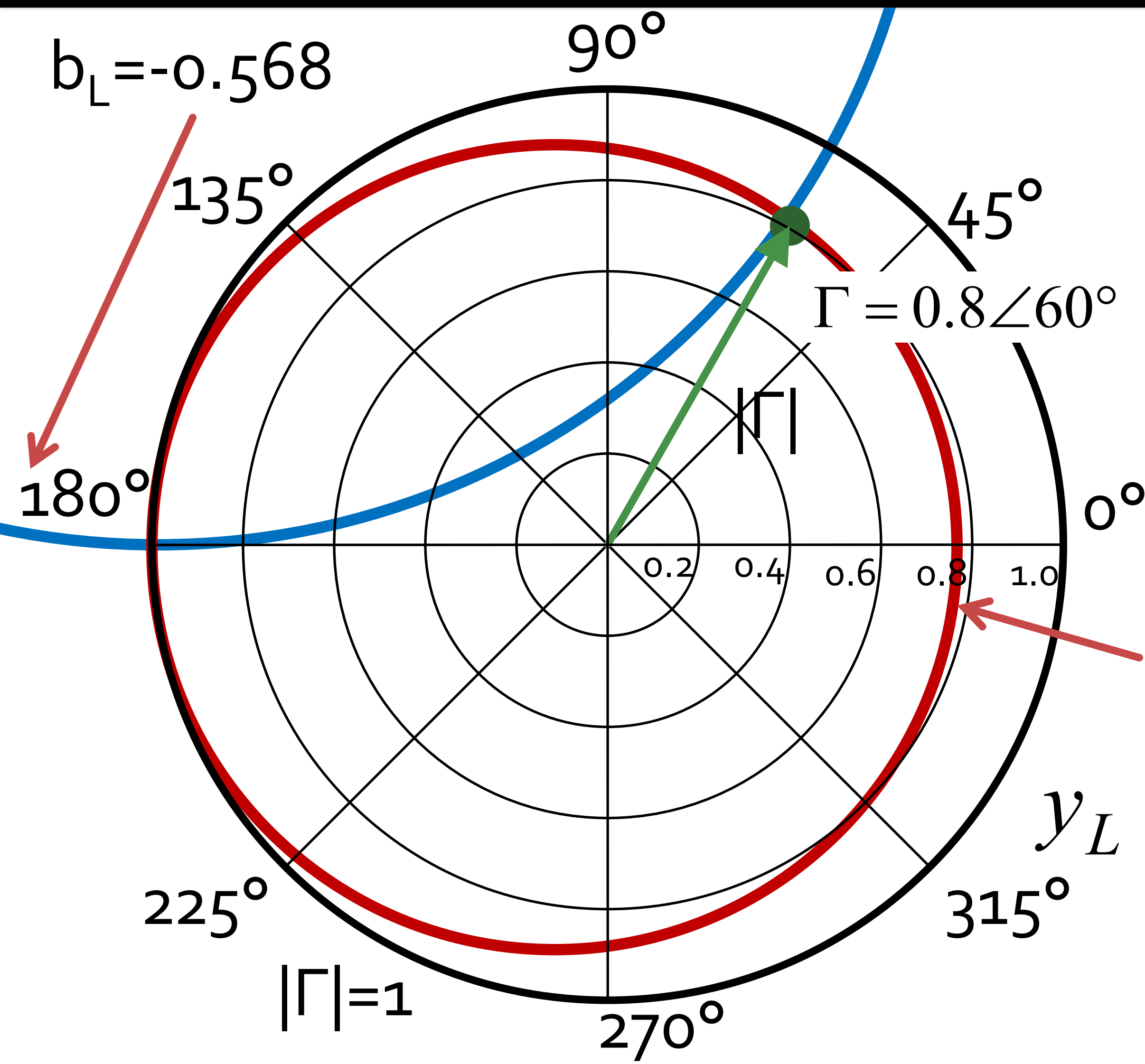
The Smith Chart, conductance



The Smith Chart, susceptance



The Smith Chart, reflection coefficient ⇔ admittance



$$\Gamma = 0.8\angle 60^\circ$$

$$Z_L = 21.429\Omega + j\cdot 82.479\Omega$$

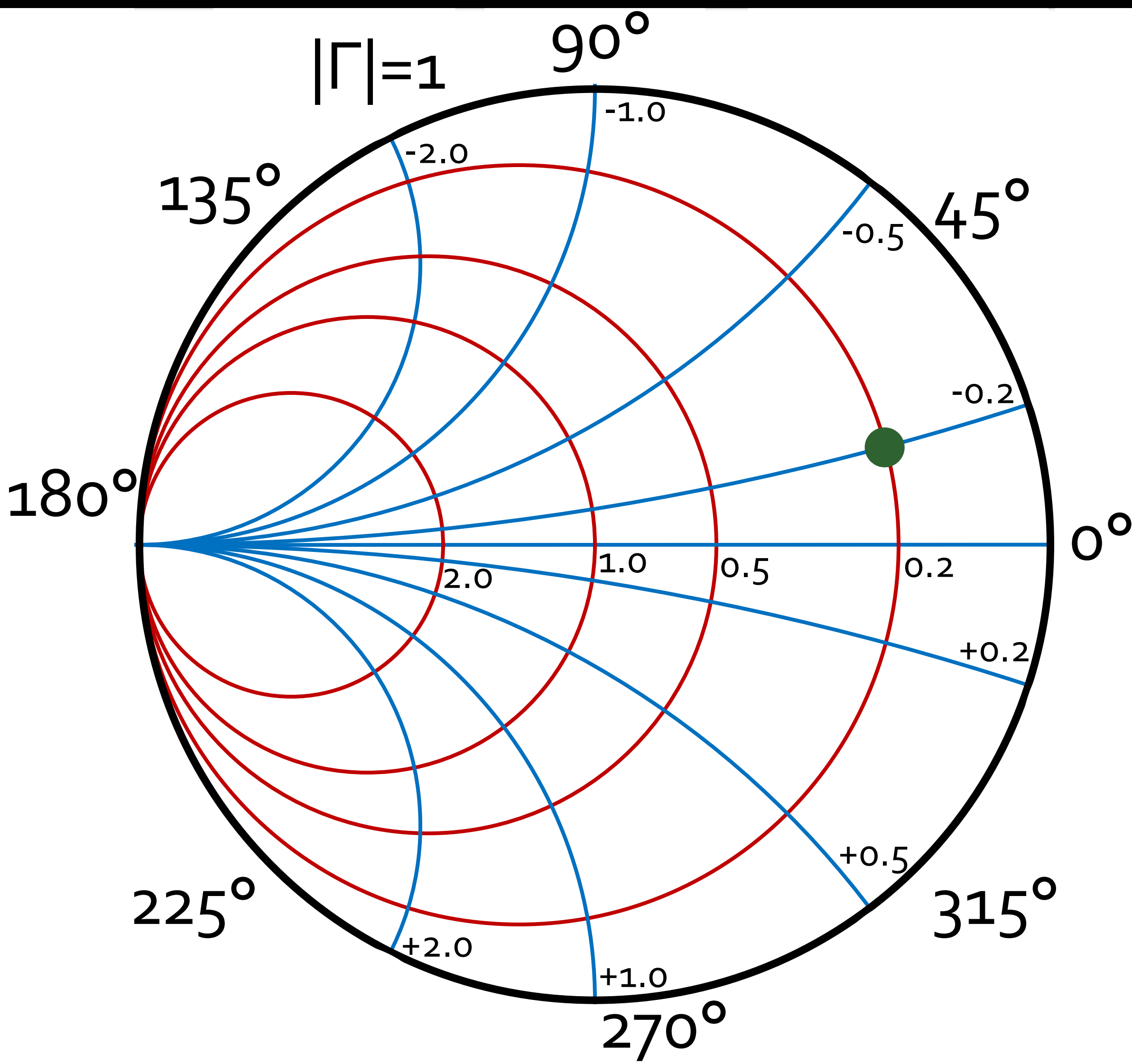
$$z_L = 0.429 + j\cdot 1.65$$

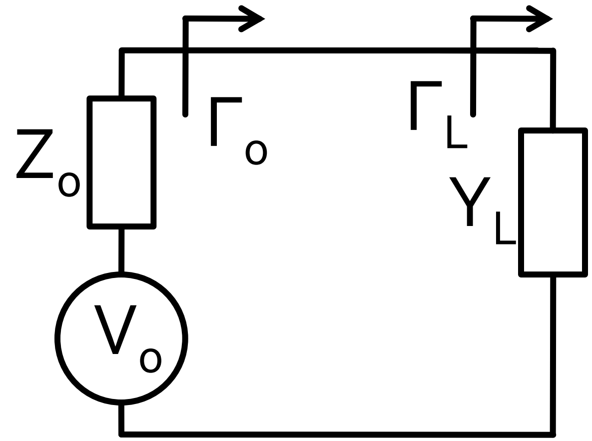
$$y_L = \frac{1}{z_L} = 0.148 - j\cdot 0.568$$

$$y_L = 0.148 + j\cdot 0.568$$

(whatever Z_o)

The Smith Chart, reflection coefficient ⇔ admittance





$$Z_0 = 50\Omega, Y_0 = 0.02S$$

$$Z_L = 125\Omega + j \cdot 125\Omega$$

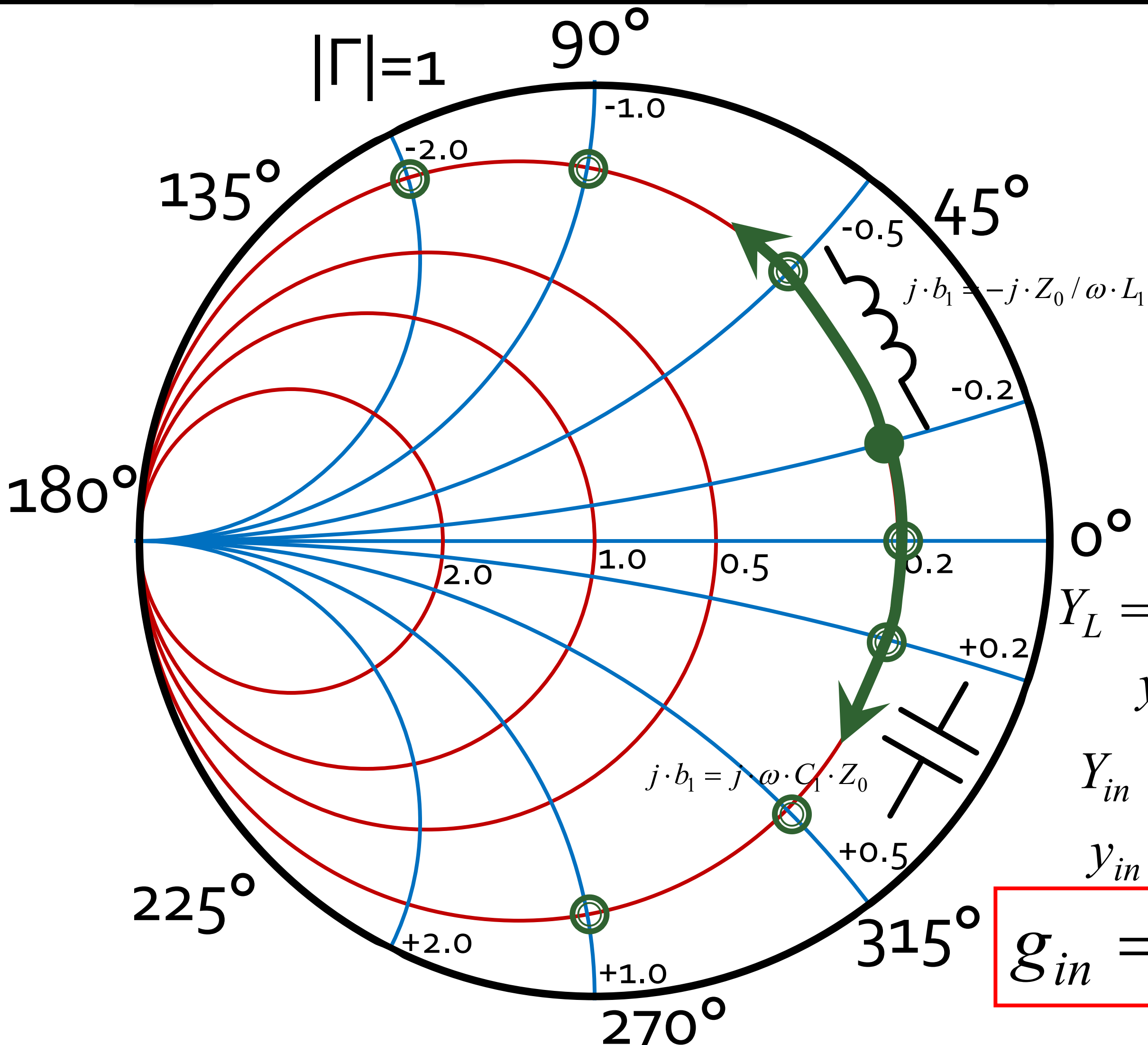
$$z_L = 2.5 + j \cdot 2.5$$

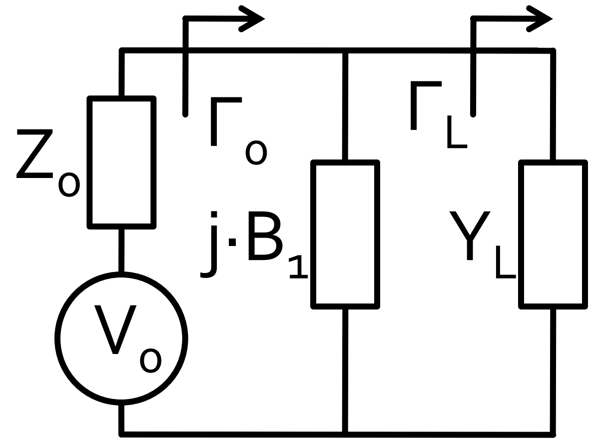
$$\Gamma_L = \Gamma_0 = 0.678\angle 23.5°$$

$$Y_L = \frac{1}{Z_L} = 0.004S - j \cdot 0.004S$$

$$y_L = \frac{1}{z_L} = \frac{Y_L}{Y_0} = 0.2 - j \cdot 0.2$$

The Smith Chart, shunt susceptance





$$Z_0 = 50\Omega, Y_0 = 0.02S$$

$$\Gamma_L = 0.678\angle 23.5°$$

$$Y_L = G_L + j \cdot B_L = 0.004S + j \cdot 0.004$$

$$y_L = g_L + j \cdot b_L = 0.2 - j \cdot 0.2$$

$$Y_{in} = Y_L + j \cdot B_1 = G_L + j \cdot (B_L + B_1)$$

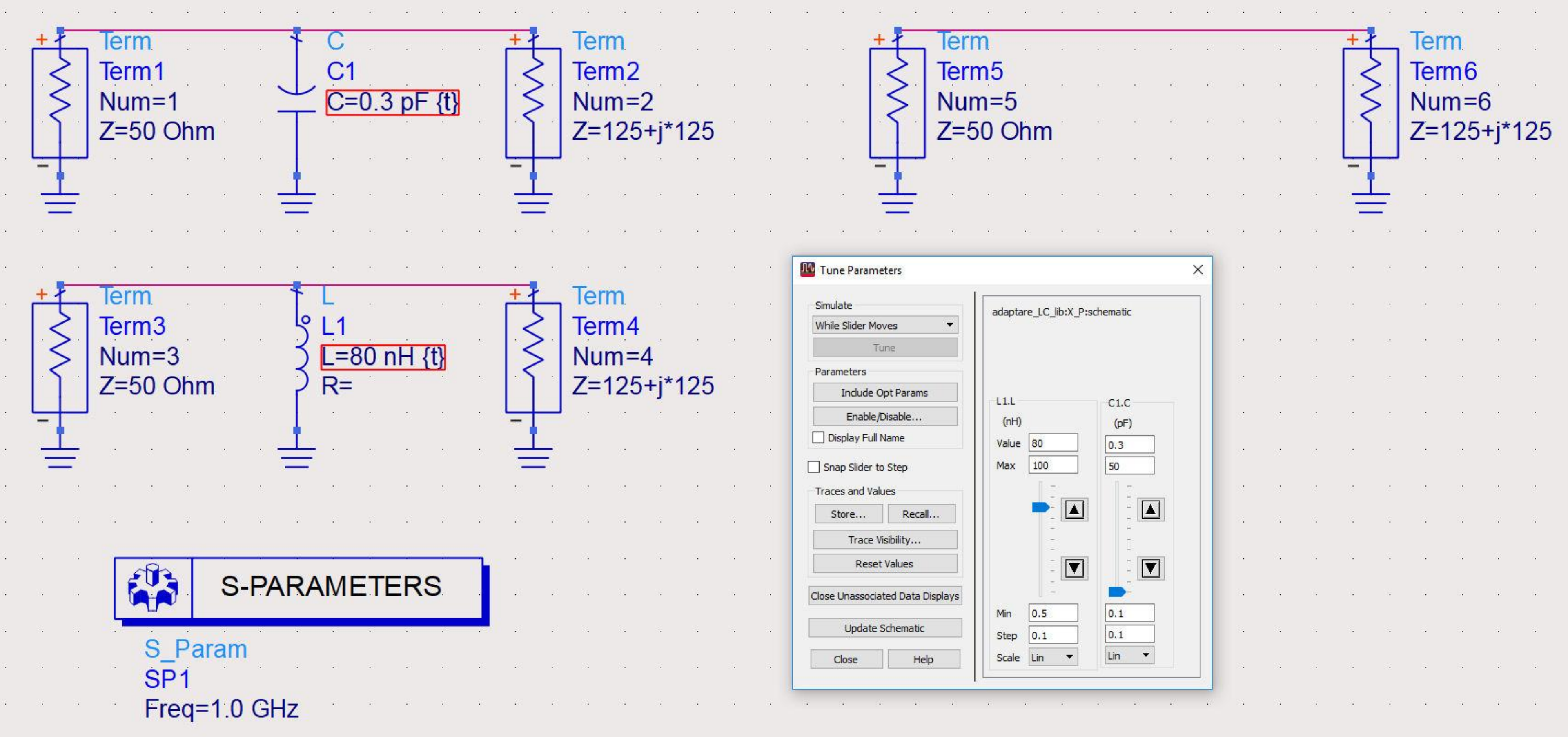
$$y_{in} = g_L + j \cdot (b_L + b_1)$$

$$g_{in} = g_L$$

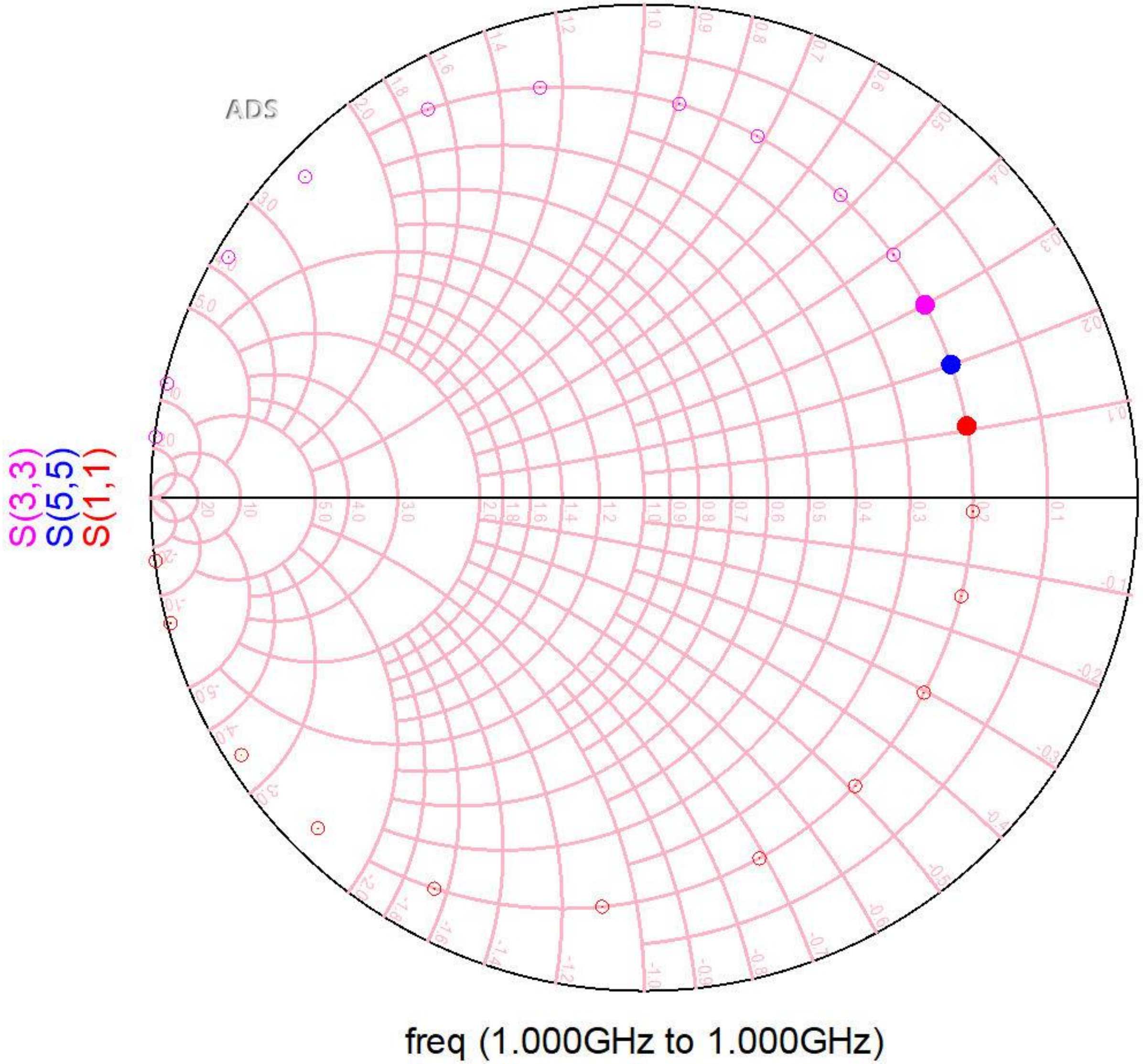
$$j \cdot b_1 = j \cdot \omega \cdot C_1 \cdot Z_0 > 0$$

$$j \cdot b_1 = -j \cdot Z_0 / \omega \cdot L_1 < 0$$

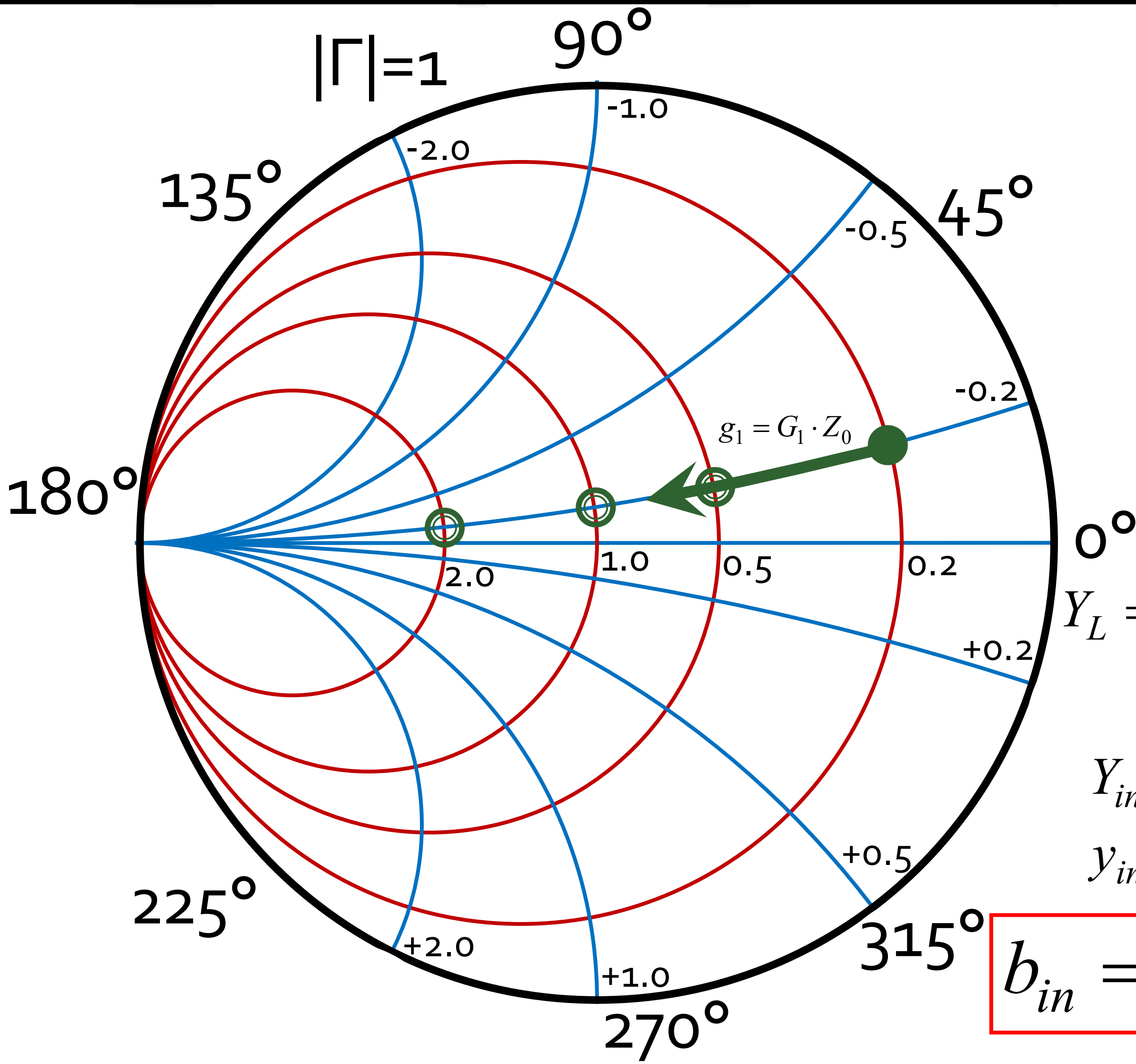
ADS, shunt susceptance

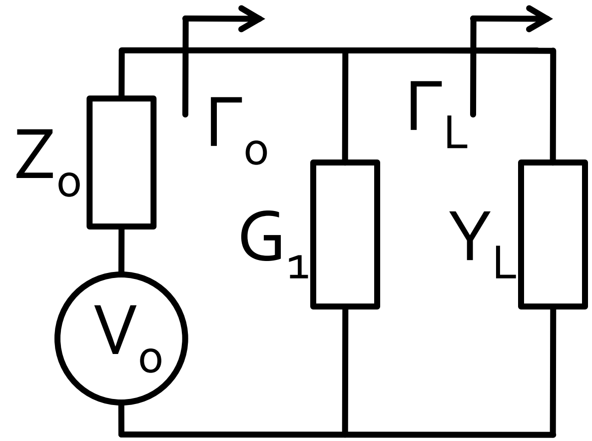


ADS, shunt susceptance



The Smith Chart, shunt conductance





$$Z_0 = 50\Omega, Y_0 = 0.02S$$

$$\Gamma_L = 0.678\angle 23.5°$$

$$Y_L = G_L + j \cdot B_L = 0.004S + j \cdot 0.004$$

$$y_L = g_L + j \cdot b_L = 0.2 - j \cdot 0.2$$

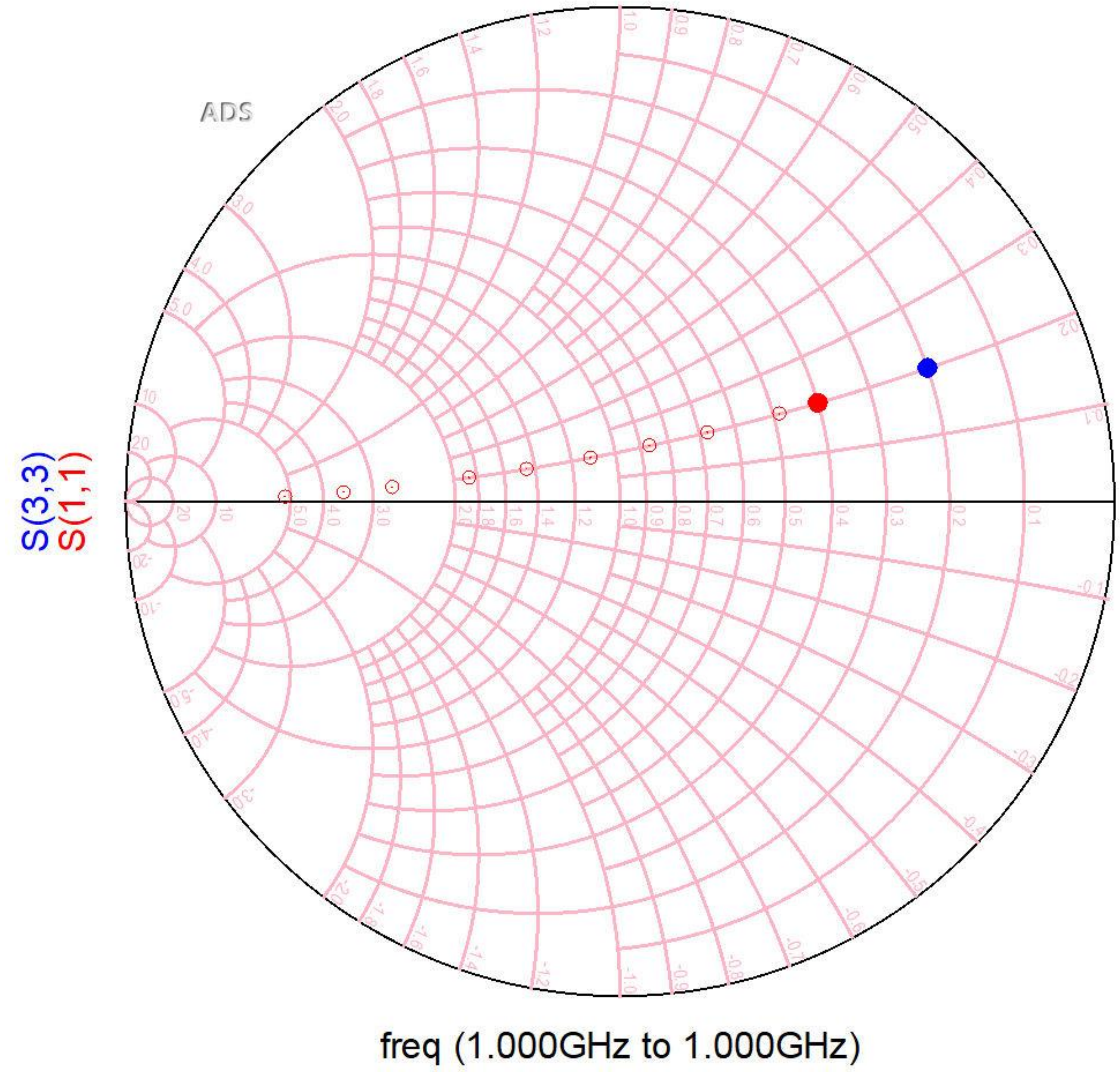
$$Y_{in} = Y_L + G_1 = (G_L + G_1) + j \cdot B_L$$

$$y_{in} = (g_L + g_1) + j \cdot b_L$$

$$b_{in} = b_L$$

$$g_{in} = g_L + G_1 \cdot Z_0$$

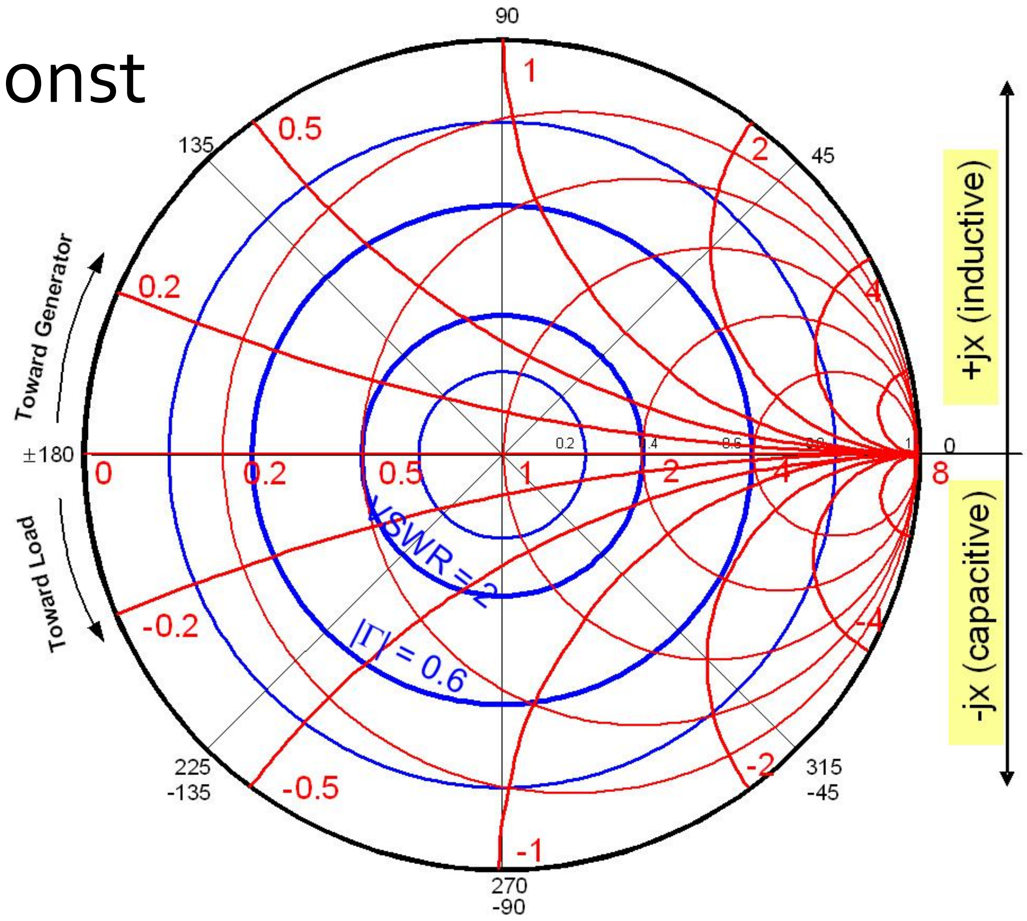
ADS, shunt conductance



Constant VSWR circles

- Certain applications may require a certain ratio between maximum / minimum line voltage
  - VSWR = const → |Γ| = const

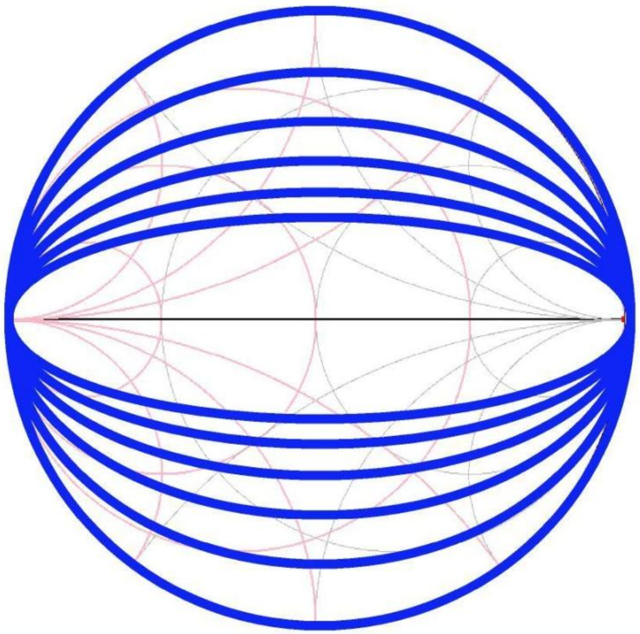
$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

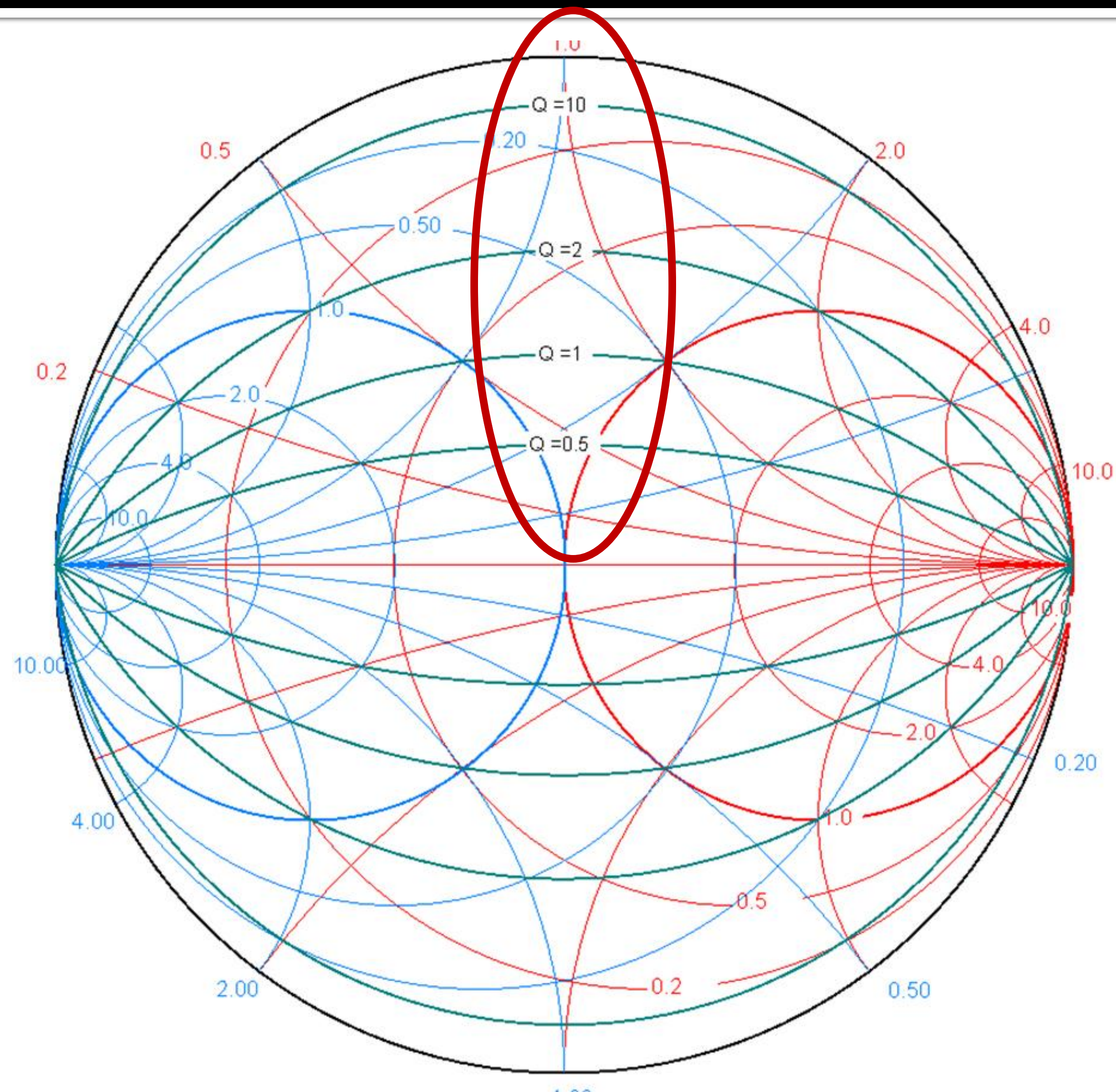


Constant Q circles

- Quality factor Q

$$Q = \frac{X}{R} = \frac{G}{B} = const$$





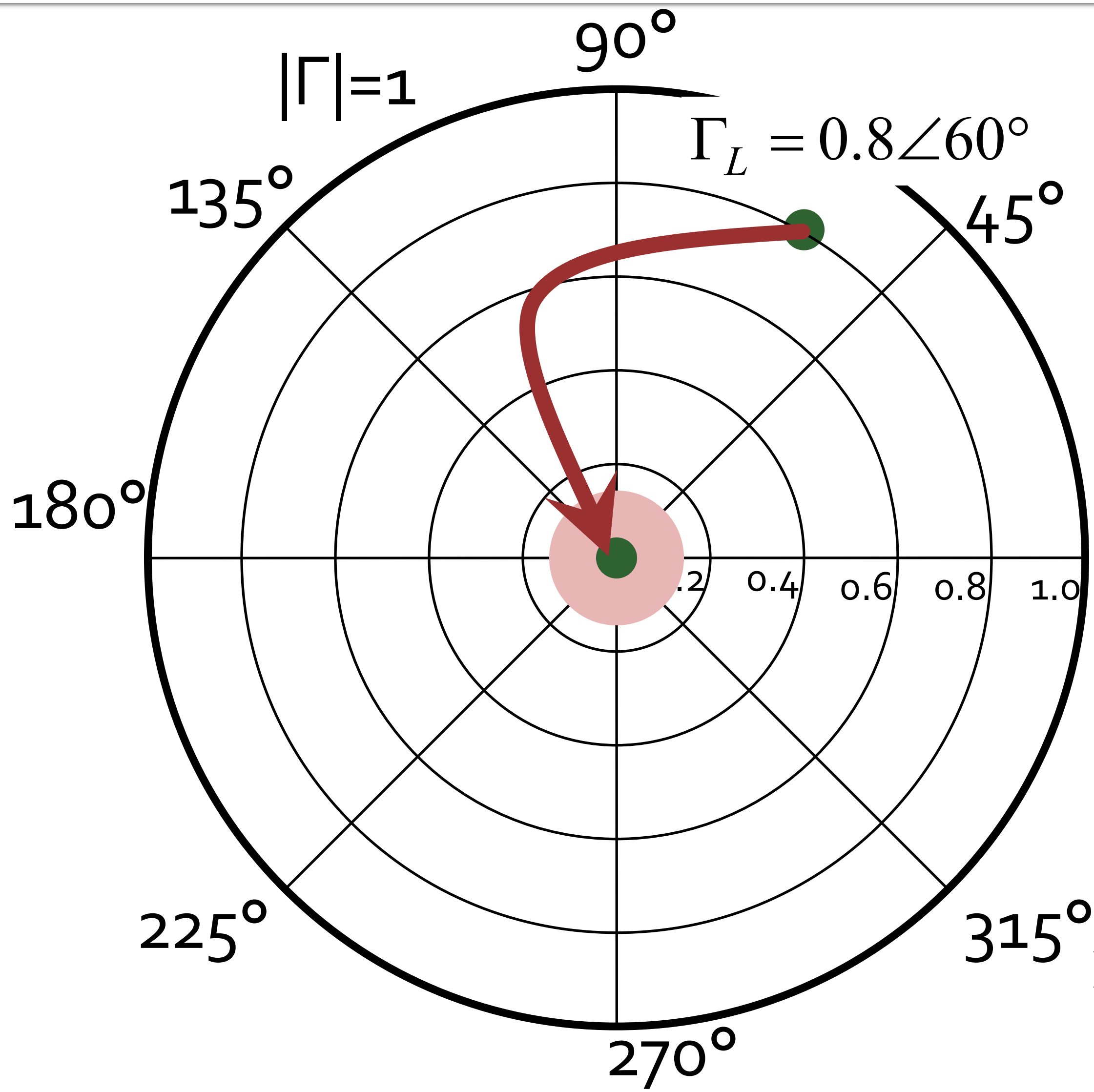
Impedance matching

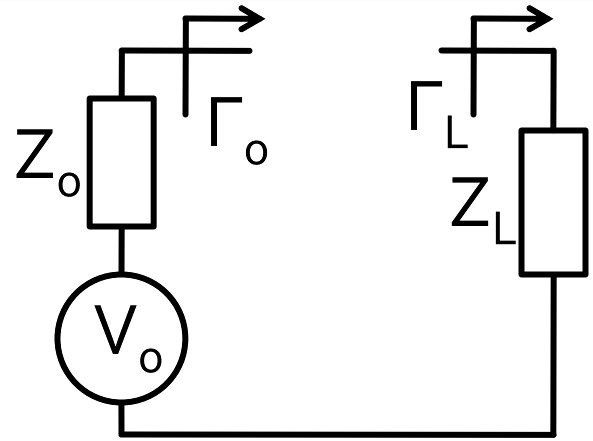
Impedance Matching with lumped elements (L Networks)

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

The Smith Chart, reflection coefficient, impedance matching





Matching Z_L load to Z_o source.
We normalize Z_L over Z_o

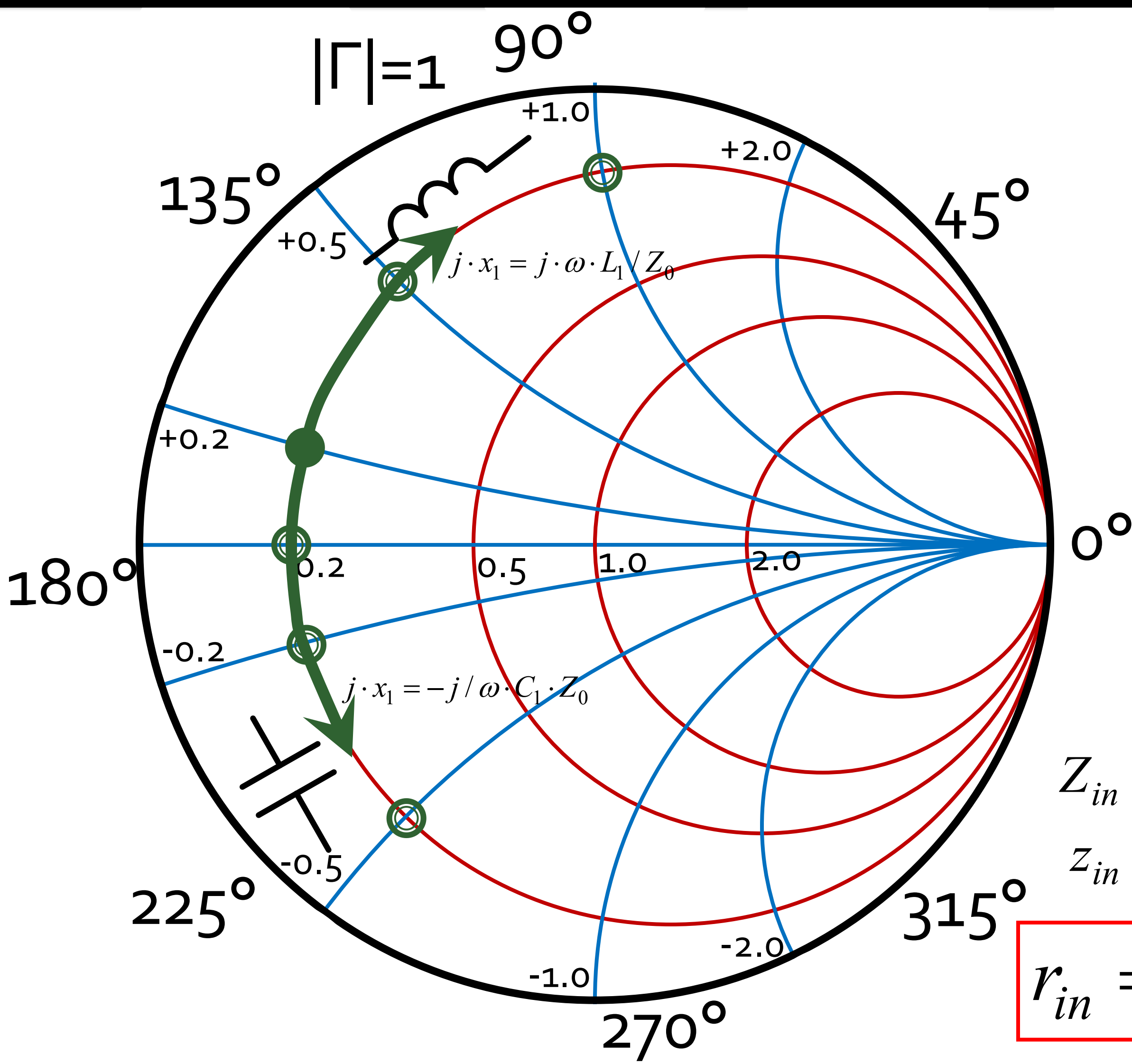
$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$
$$z_L = 0.429 + j \cdot 1.65$$
$$\Gamma_L = 0.8\angle 60°$$

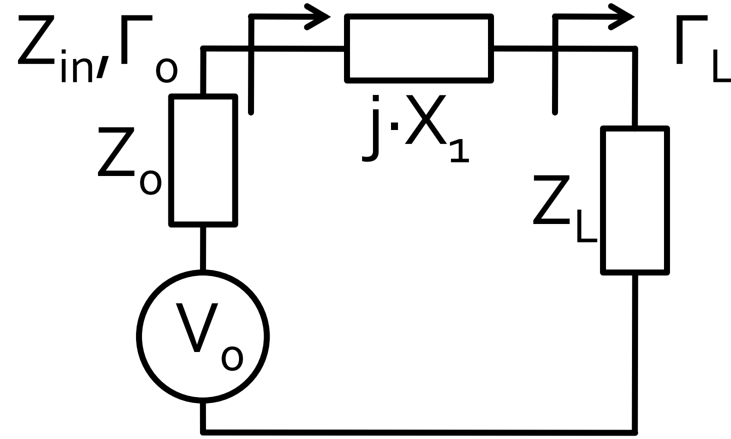
We must move the point denoting the reflection coefficient in the area where with a Z_o source we have:

$\Gamma_0 = 0$ perfect match

$|\Gamma_0| \le \Gamma_m$ "good enough" match

The Smith Chart, series reactance





$$Z_0 = 50\Omega$$

$$Z_L = R_L + j \cdot X_L = 10\Omega + j \cdot 10\Omega$$

$$z_L = r_L + j \cdot x_L = 0.2 + j \cdot 0.2$$

$$\Gamma_L = 0.678\angle 156.5°$$

$$Z_{in} = Z_L + j \cdot X_1 = R_L + j \cdot (X_L + X_1)$$

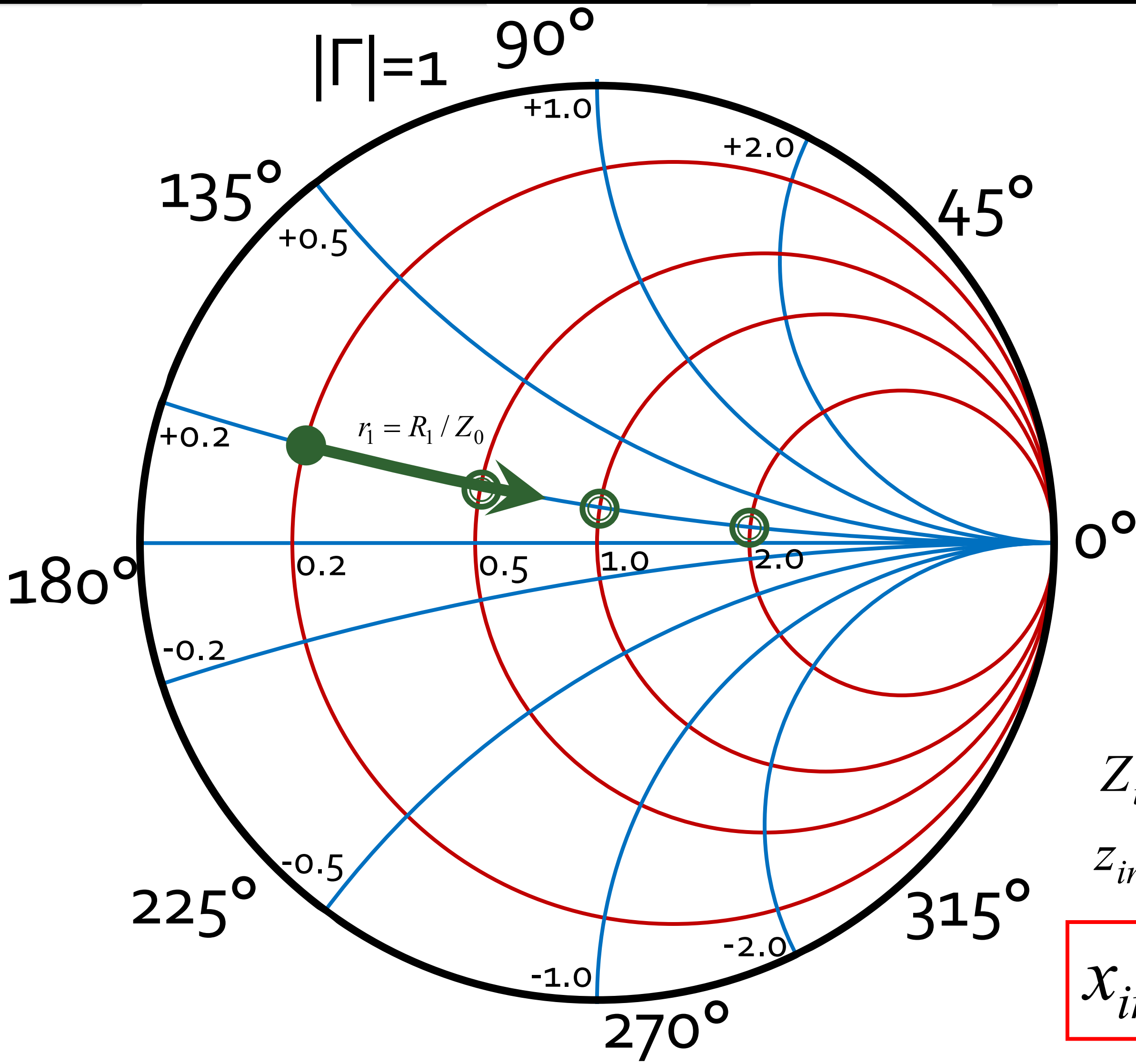
$$z_{in} = r_L + j \cdot (x_L + x_1)$$

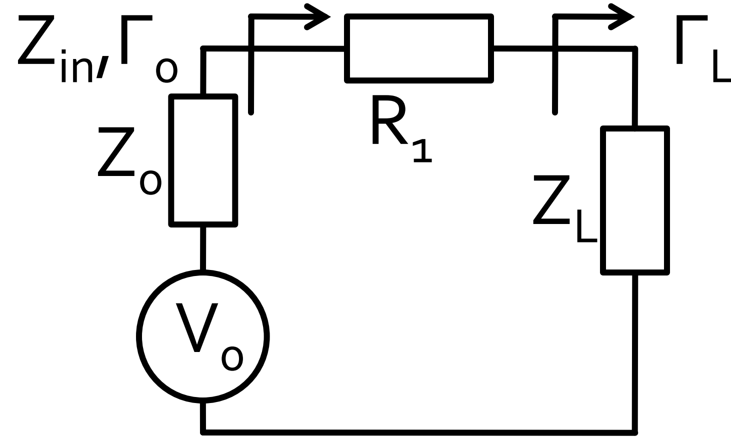
$$r_{in} = r_L$$

$$j \cdot x_1 = j \cdot \omega \cdot L_1 / Z_0 > 0$$

$$j \cdot x_1 = -j / \omega \cdot C_1 \cdot Z_0 < 0$$

The Smith Chart, series resistance





$$Z_0 = 50\Omega$$

$$Z_L = R_L + j \cdot X_L = 10\Omega + j \cdot 10$$

$$z_L = r_L + j \cdot x_L = 0.2 + j \cdot 0.2$$

$$\Gamma_L = 0.678\angle 156.5°$$

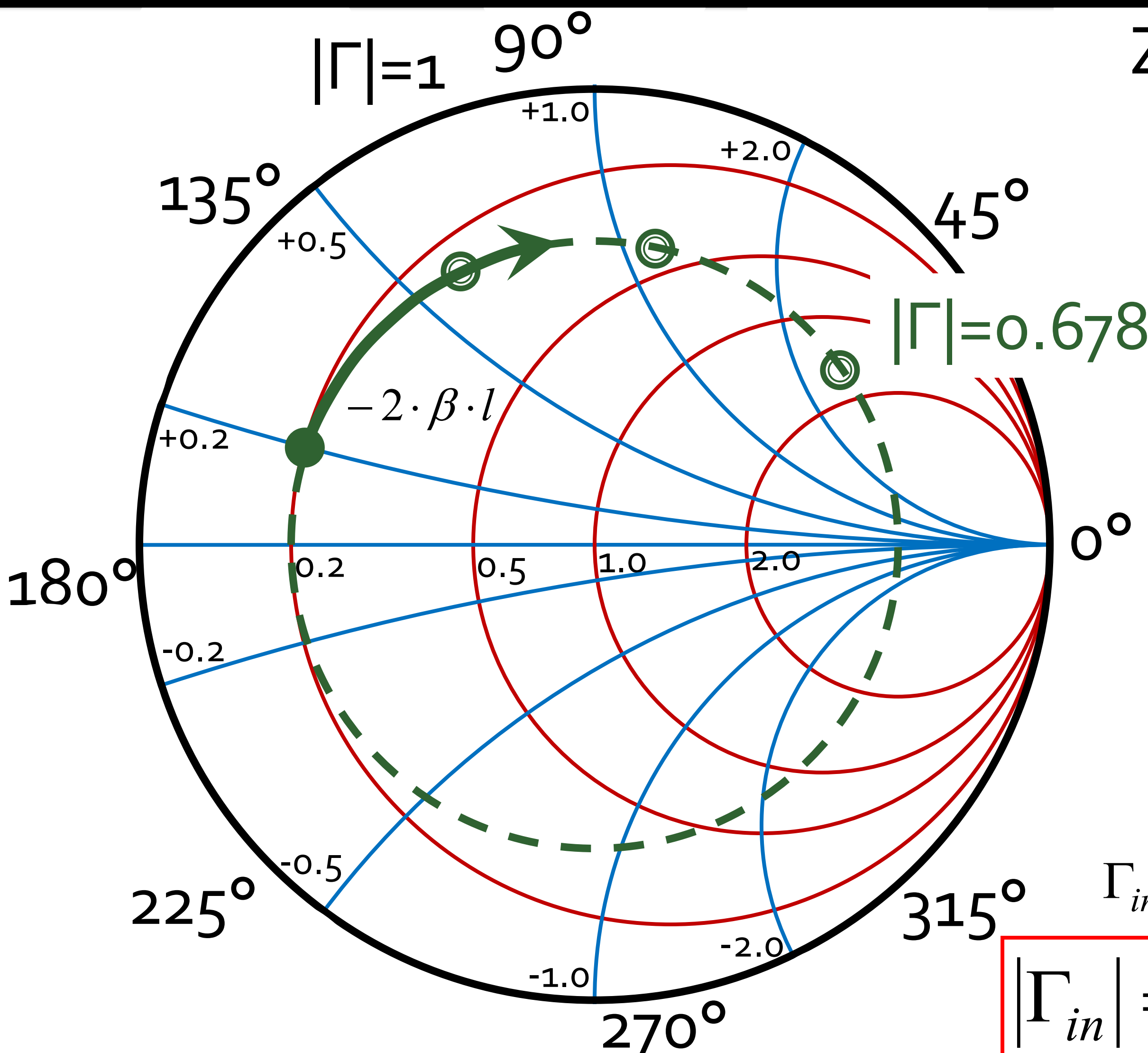
$$Z_{in} = Z_L + R_1 = (R_L + R_1) + j \cdot X_L$$

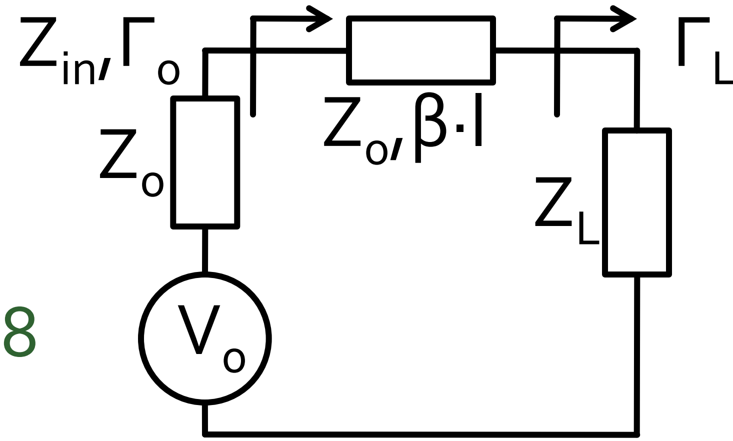
$$z_{in} = z_L + r_1 = (r_L + r_1) + j \cdot x_L$$

$$\boxed{x_{in} = x_L}$$

$$r_{in} = r_L + R_1 / Z_0$$

The Smith Chart, series transmission line, Z_o





$Z_0 = 50\Omega$

$Z_L = R_L + j \cdot X_L = 10\Omega + j \cdot 10$...

$z_L = r_L + j \cdot x_L = 0.2 + j \cdot 0.2$

$\Gamma_L = 0.678\angle 156.5°$

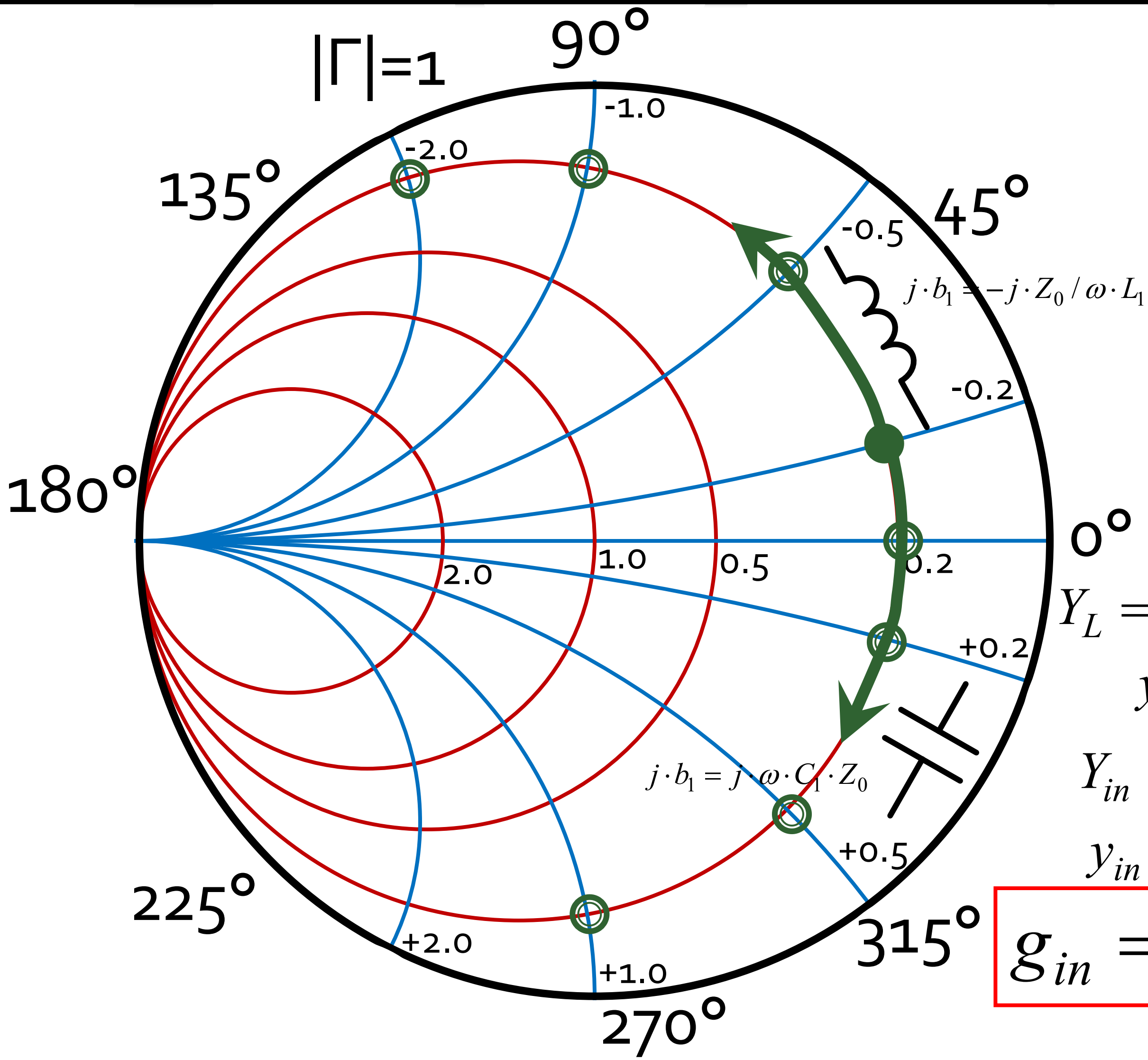
$$Z_{in} = Z_0 \cdot \frac{1+\Gamma_L \cdot e^{-2j\cdot\beta\cdot l}}{1-\Gamma_L \cdot e^{-2j\cdot\beta\cdot l}}$$

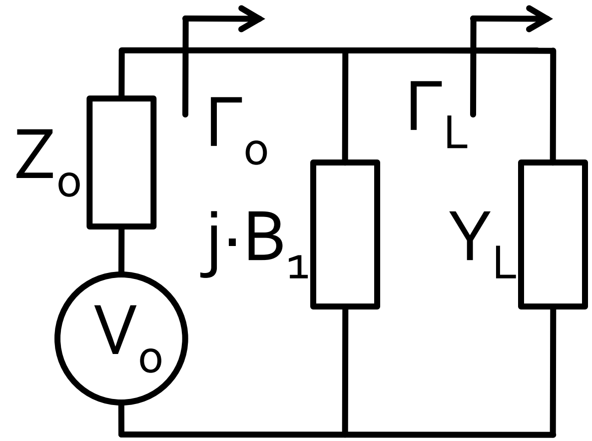
$\Gamma_{in} = \Gamma_L \cdot e^{-2j\cdot\beta\cdot l}$

$$|\Gamma_{in}| = |\Gamma_L|$$

$\arg(\Gamma_{in}) = \arg(\Gamma_L) - 2 \cdot \beta \cdot$...

The Smith Chart, shunt susceptance





$$Z_0 = 50\Omega, Y_0 = 0.02S$$

$$\Gamma_L = 0.678\angle 23.5°$$

$$Y_L = G_L + j \cdot B_L = 0.004S + j \cdot 0.004$$

$$y_L = g_L + j \cdot b_L = 0.2 - j \cdot 0.2$$

$$Y_{in} = Y_L + j \cdot B_1 = G_L + j \cdot (B_L + B_1)$$

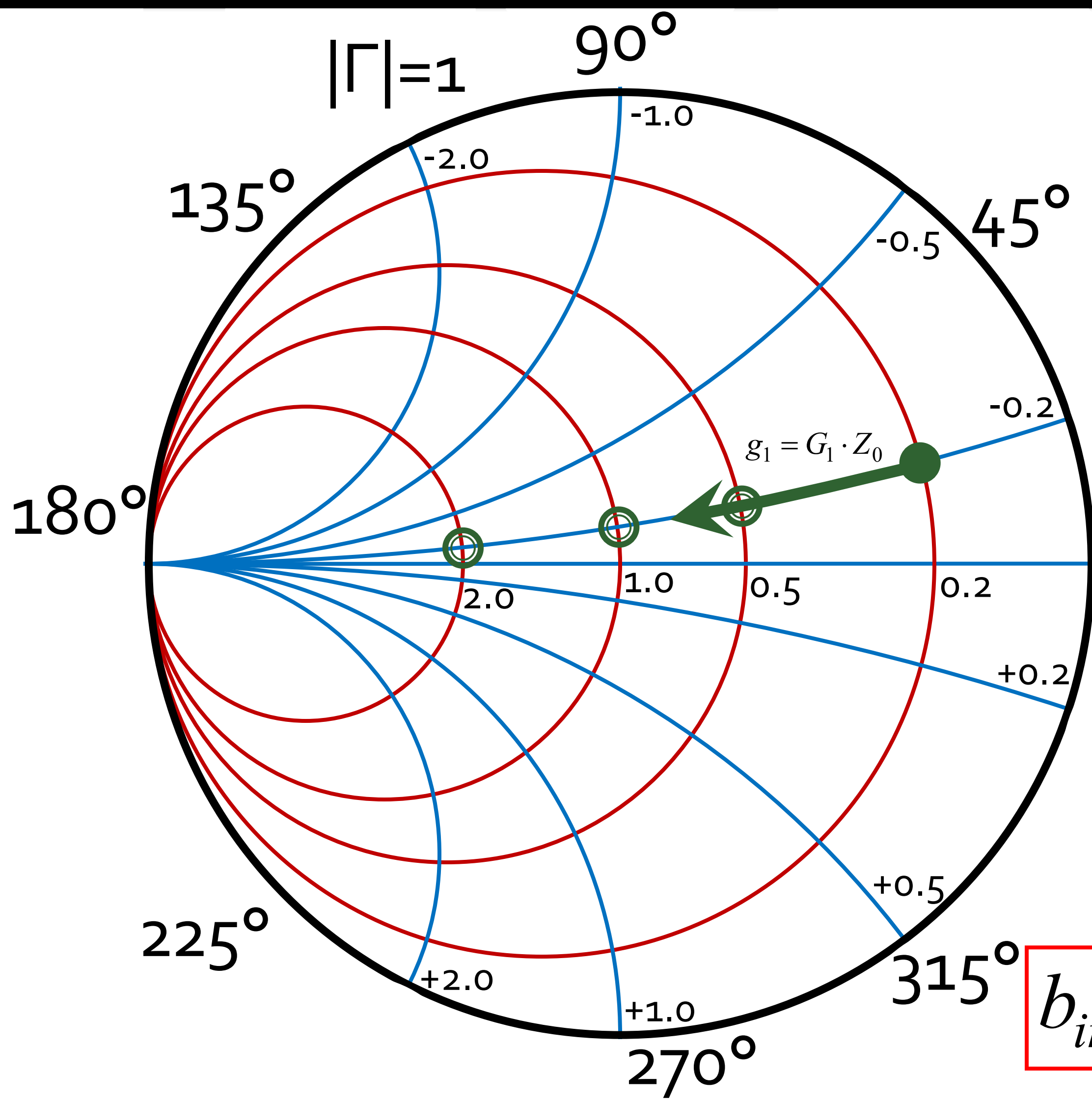
$$y_{in} = g_L + j \cdot (b_L + b_1)$$

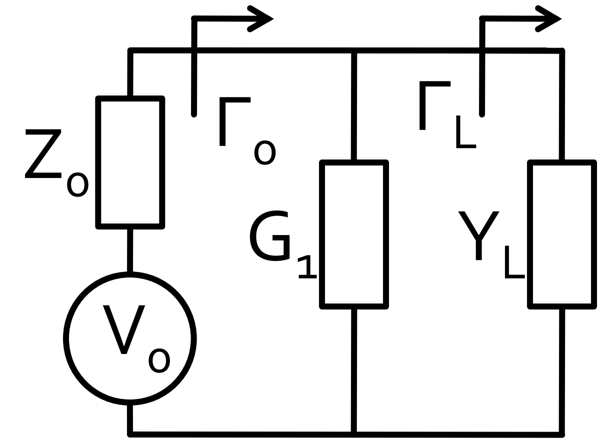
$$g_{in} = g_L$$

$$j \cdot b_1 = j \cdot \omega \cdot C_1 \cdot Z_0 > 0$$

$$j \cdot b_1 = -j \cdot Z_0 / \omega \cdot L_1 < 0$$

The Smith Chart, shunt conductance





$$Z_0 = 50\Omega, Y_0 = 0.02S$$

$$\Gamma_L = 0.678\angle 23.5°$$

$$Y_L = G_L + j \cdot B_L = 0.004S + j \cdot 0.004$$

$$y_L = g_L + j \cdot b_L = 0.2 - j \cdot 0.2$$

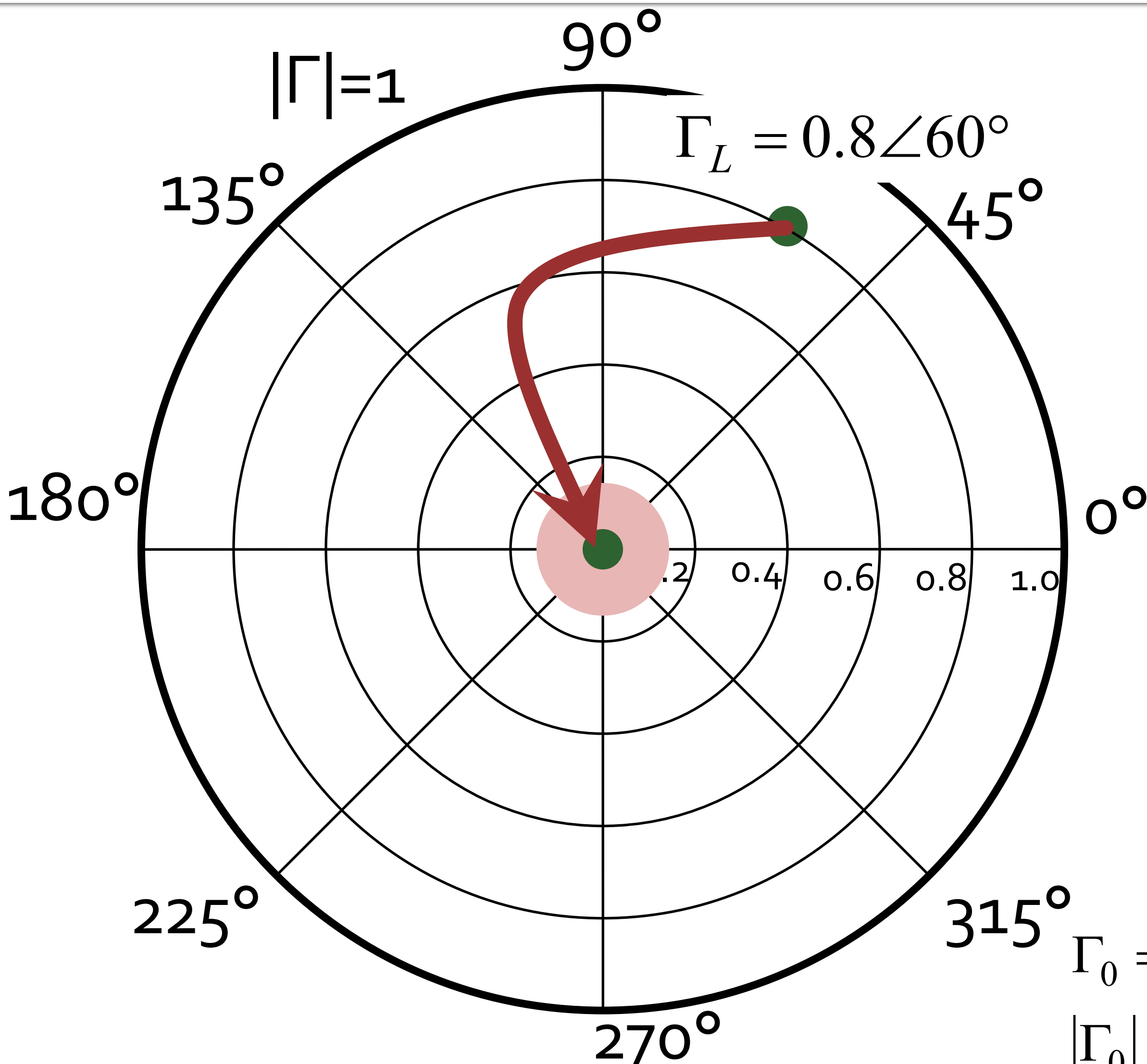
$$Y_{in} = Y_L + G_1 = (G_L + G_1) + j \cdot B_L$$

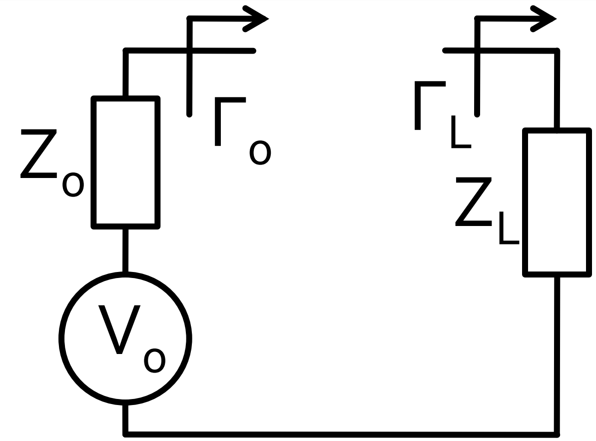
$$y_{in} = (g_L + g_1) + j \cdot b_L$$

$$b_{in} = b_L$$

$$g_{in} = g_L + G_1 \cdot Z_0$$

Impedance matching



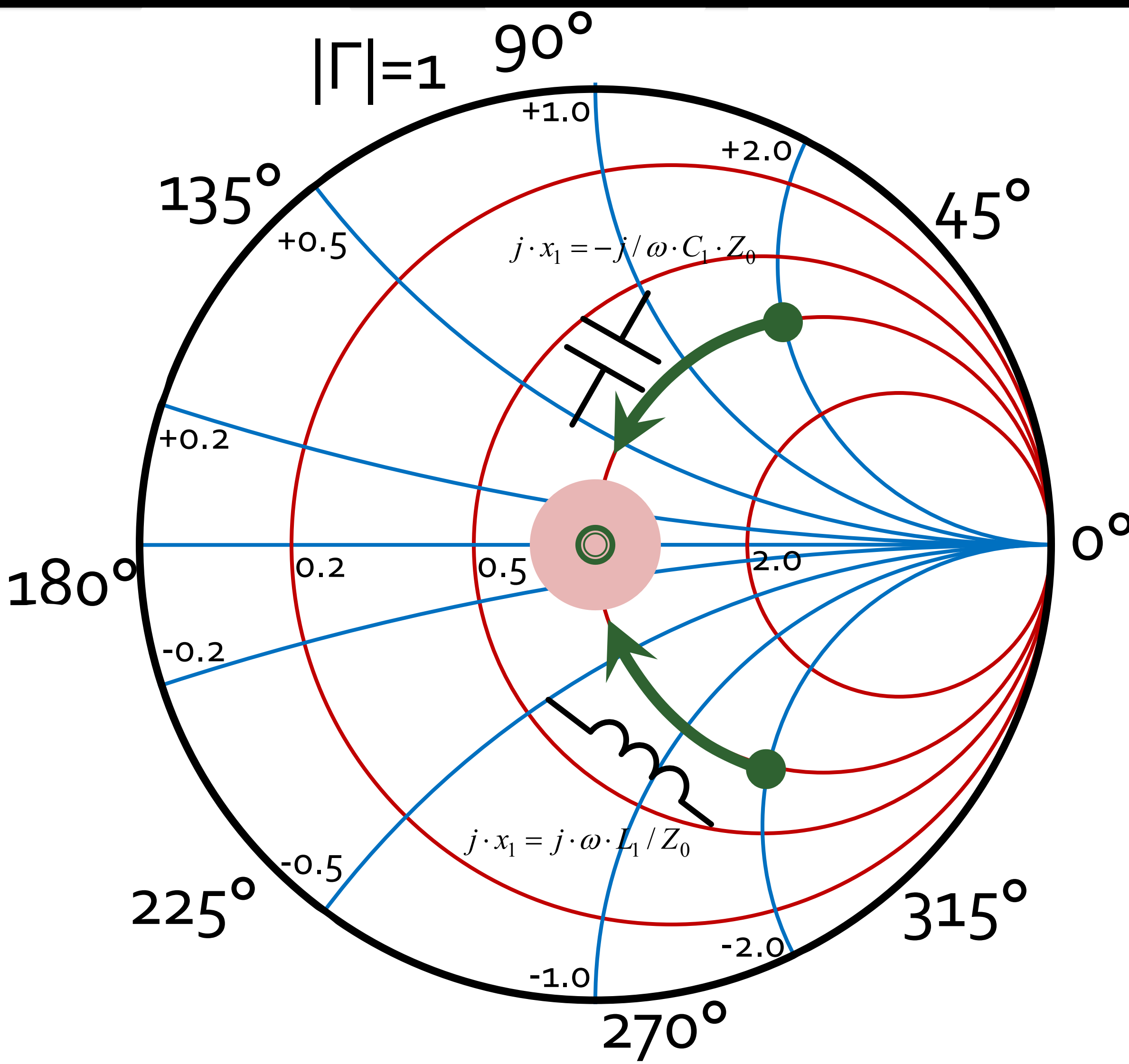


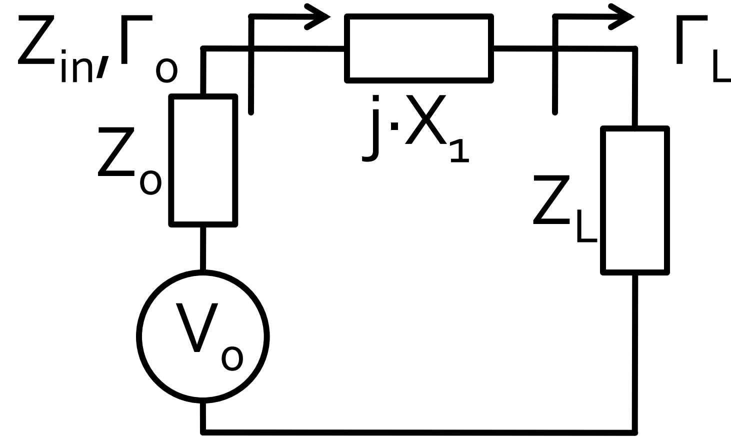
How?

$\Gamma_0 = 0$ perfect match

$|\Gamma_0| \le \Gamma_m$ "good enough" match

Matching, series reactance





$$z_L = r_L + j \cdot x_L$$

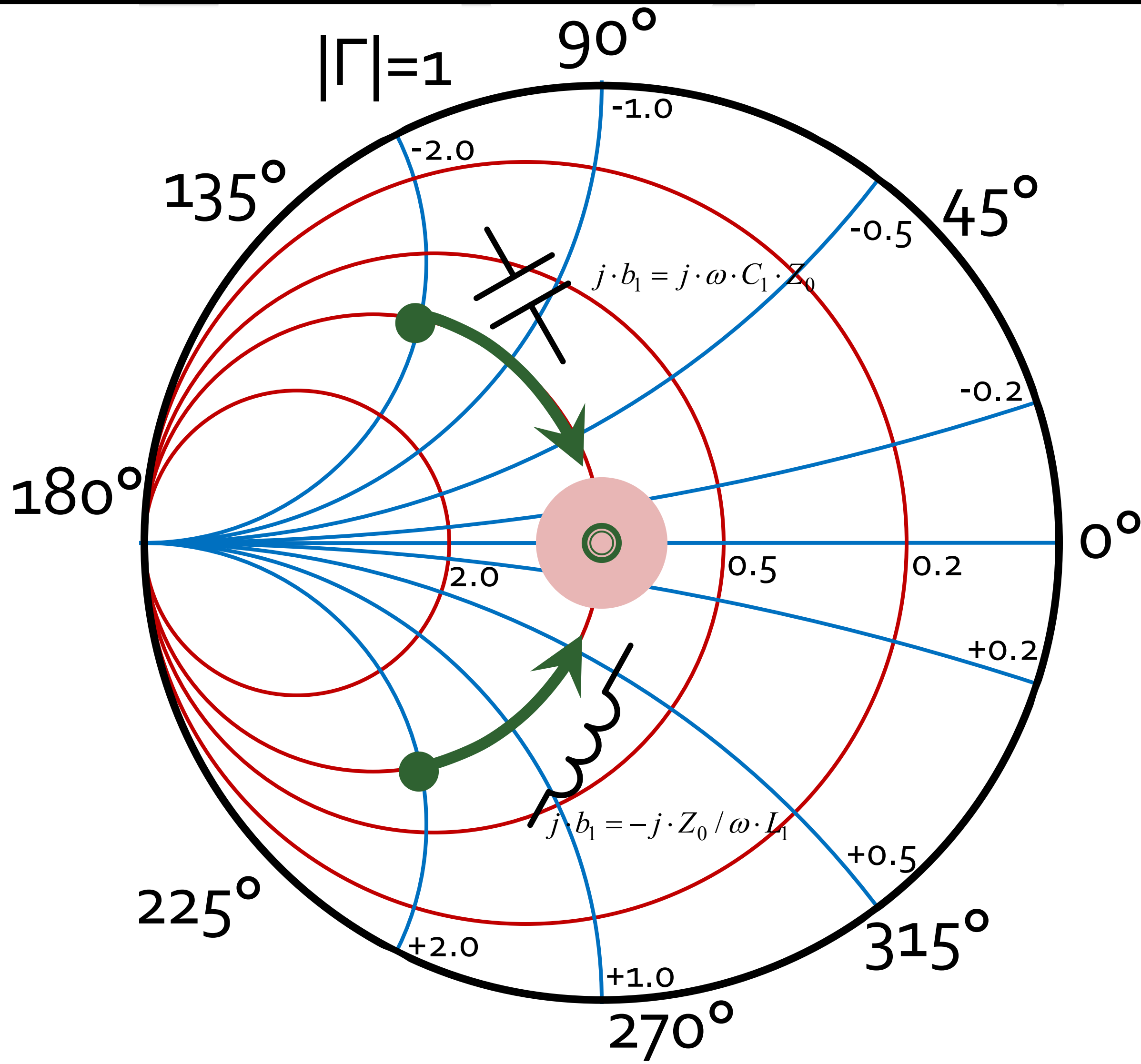
$$z_{in} = r_L + j \cdot (x_L + x_1)$$

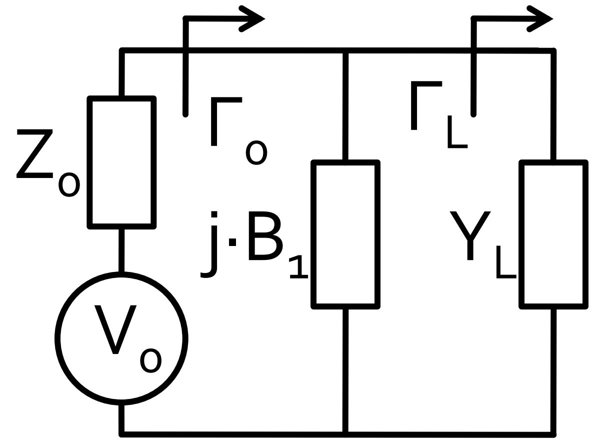
$$r_{in} = r_L$$

- Match can be obtained **if and only if** $r_L = 1$
- we compensate the reactive part of the load

$$j \cdot x_1 = -j \cdot x_L$$

Matching, shunt susceptance





$$y_L = g_L + j \cdot b_L$$

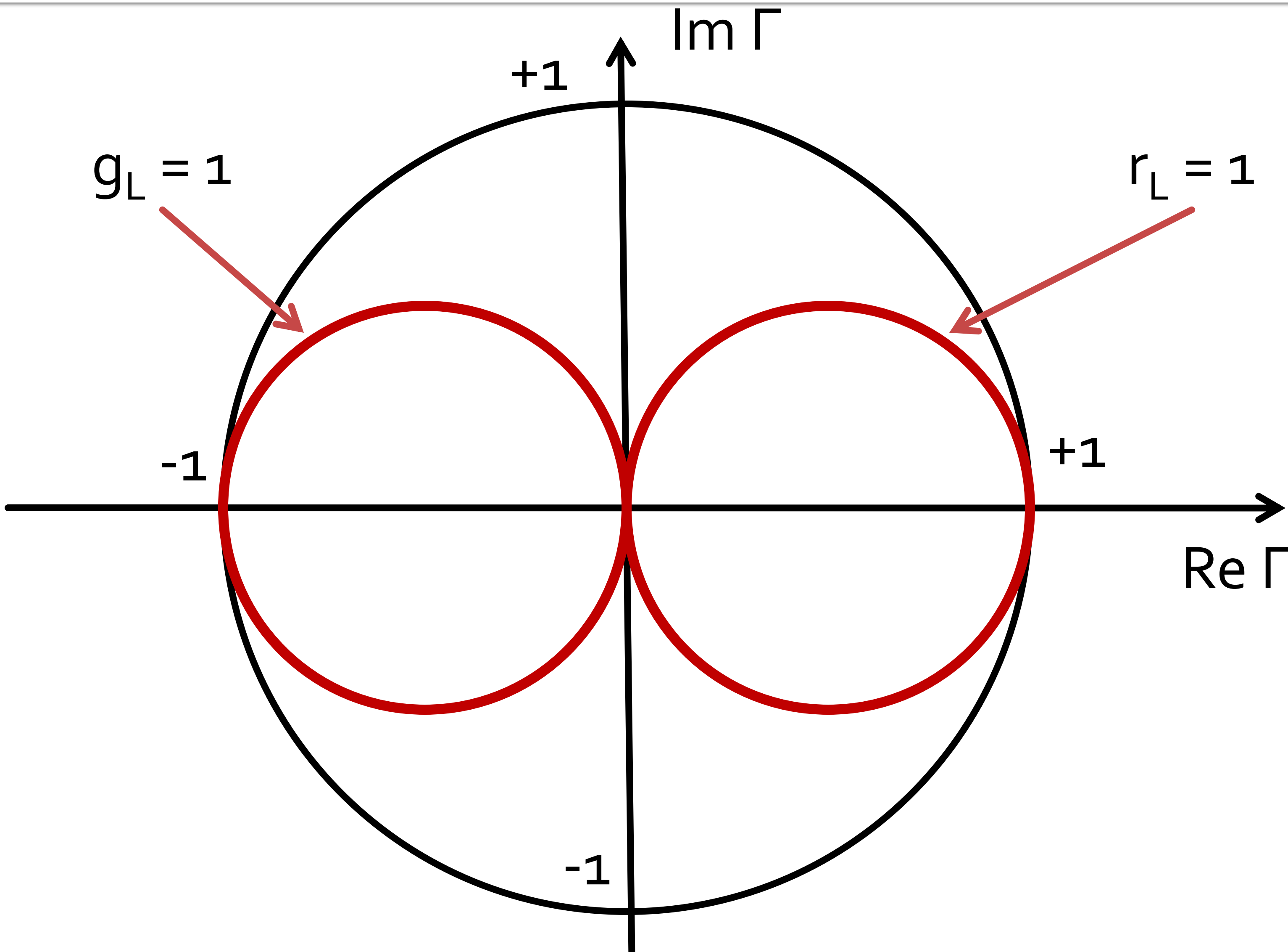
$$y_{in} = g_L + j \cdot (b_L + b_1)$$

$$g_{in} = g_L$$

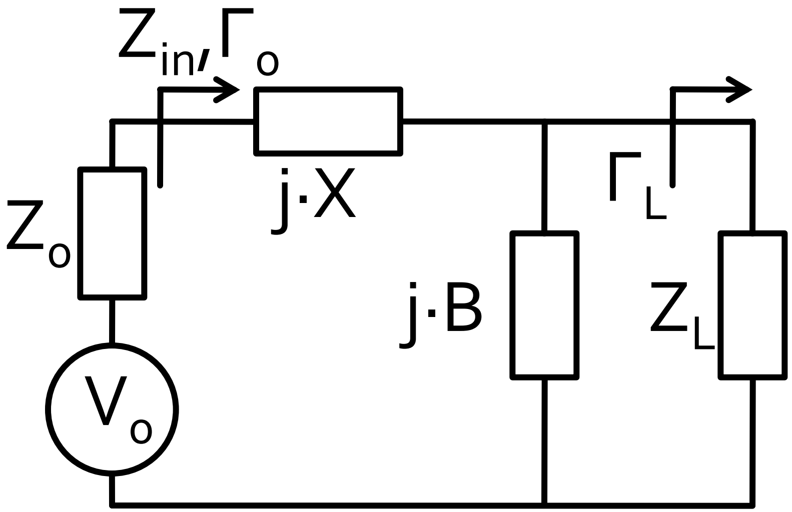
- Match can be obtained **if and only if** $g_L = 1$
- we compensate the reactive part of the load

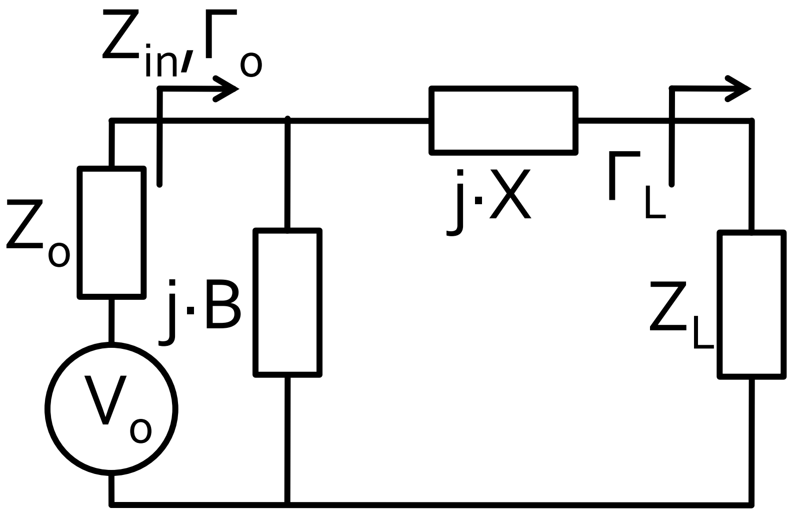
$$j \cdot b_1 = -j \cdot b_L$$

Smith chart, r=1 and g=1



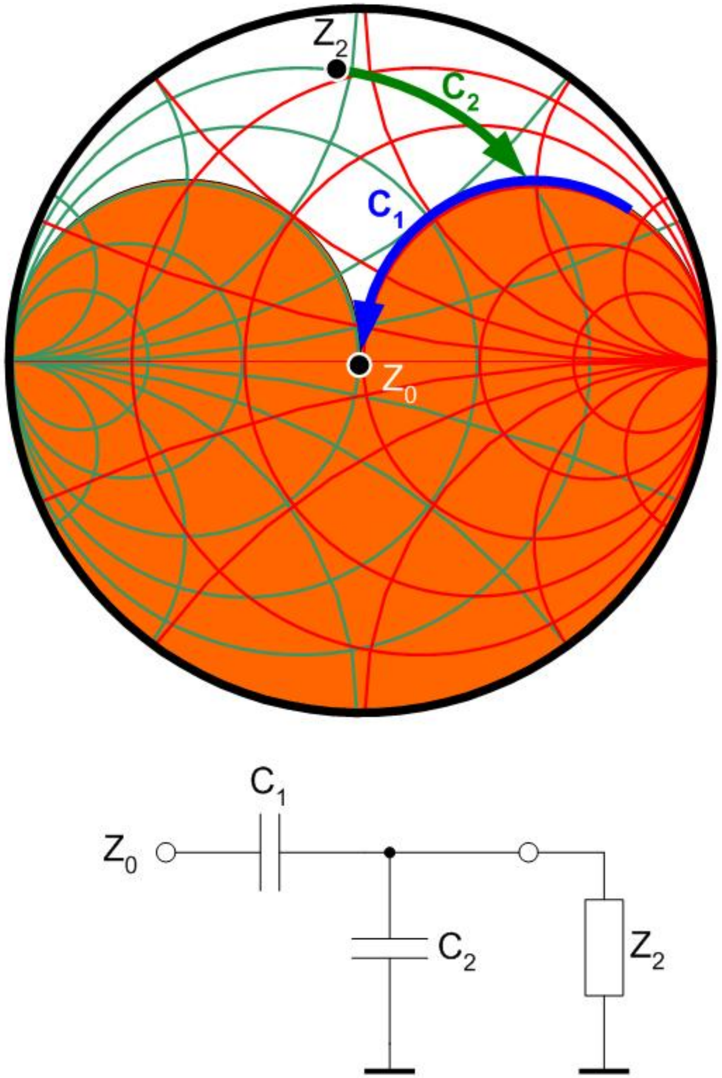
Matching with 2 reactive elements (L Networks)

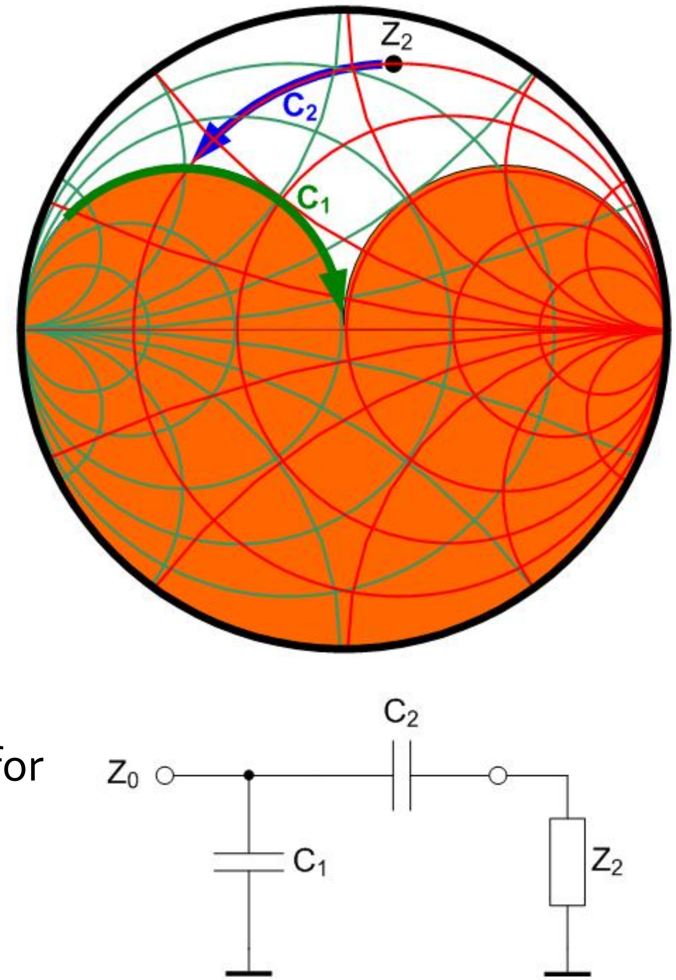




- Two steps matching
  - first reactive element moves the reflection coefficient **on the circle** $r_L = 1/g_L = 1$
  - second element compensates the remaining reactance and achieves the impedance match

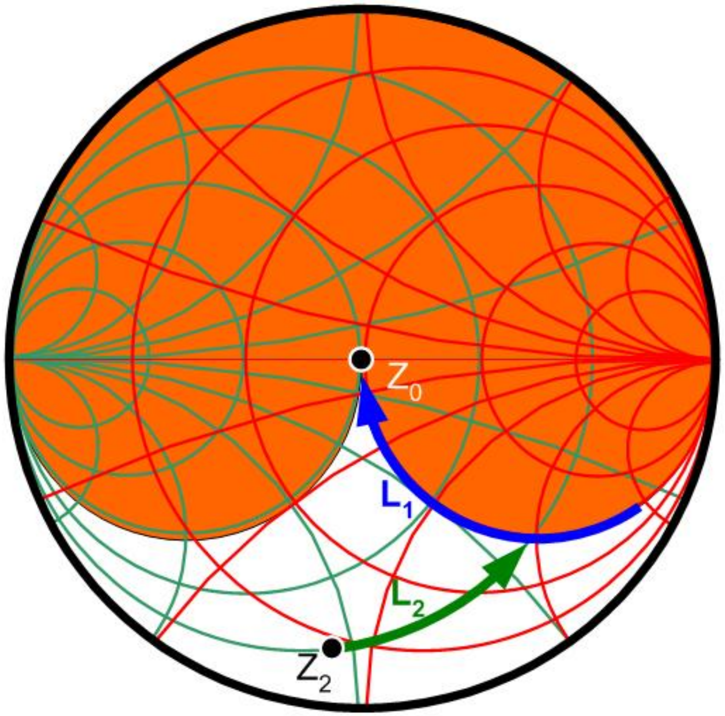
series C, shunt C / shunt C, series C

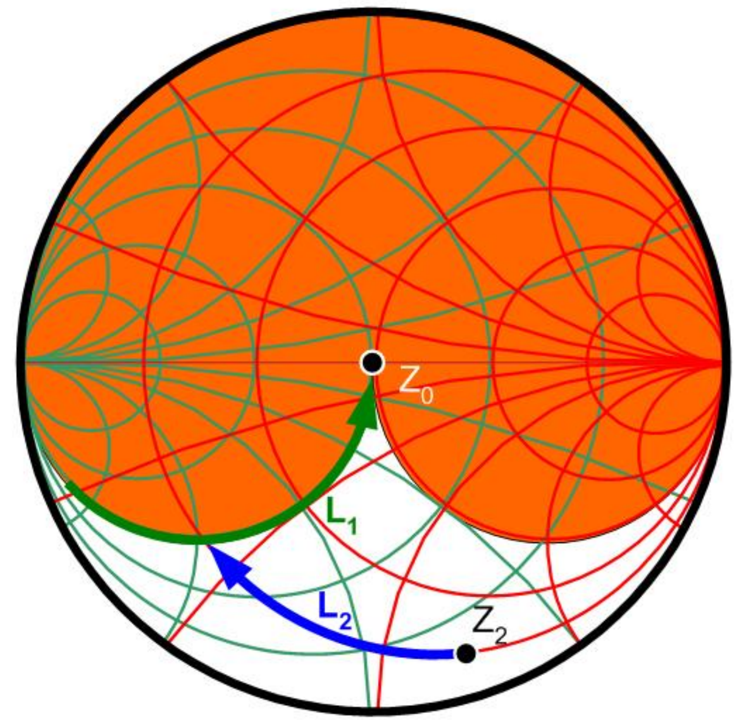


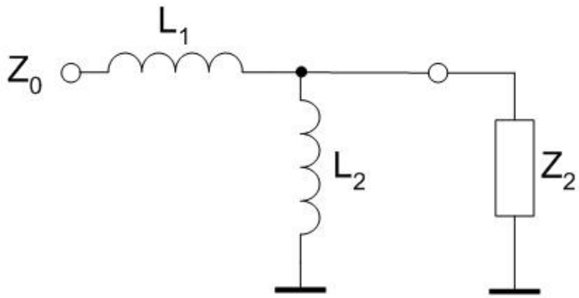


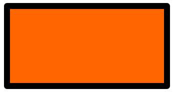
Forbidden area for current network

series L, shunt L / shunt L, series L

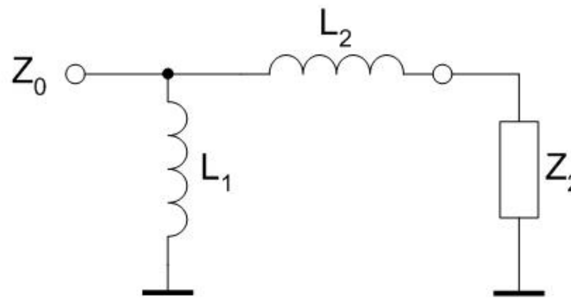




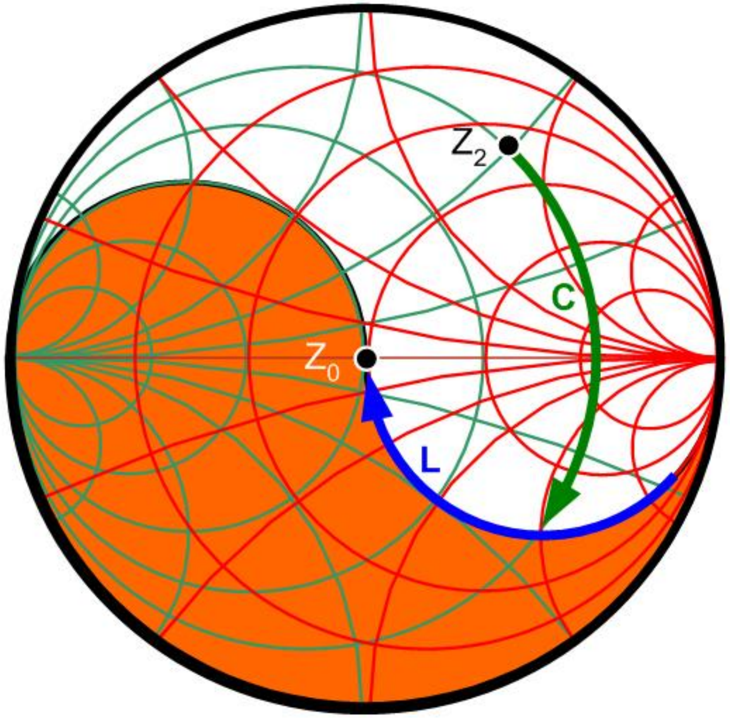


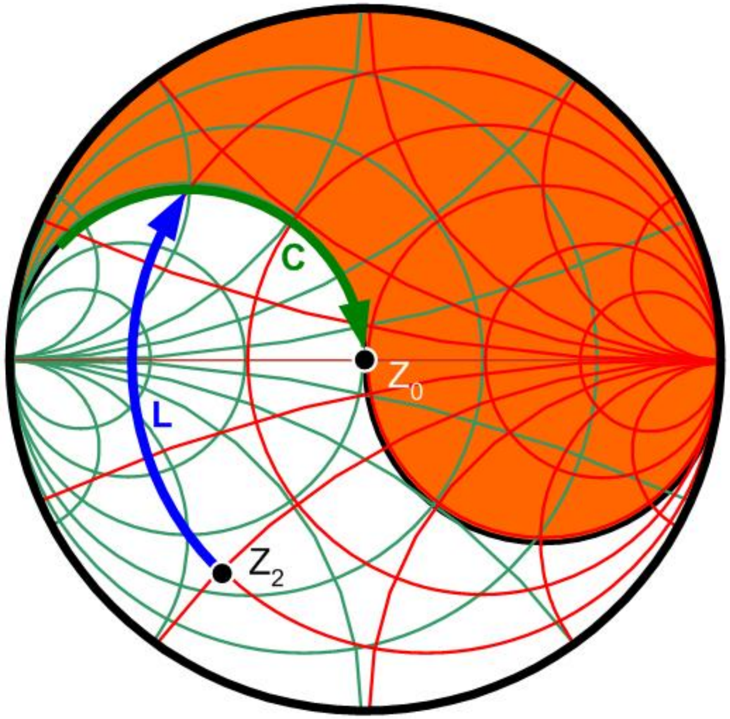


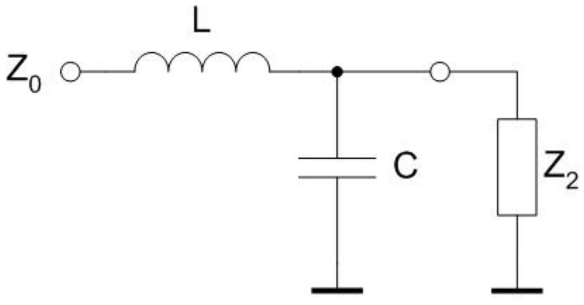
Forbidden area for current network

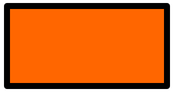


series L, shunt C / shunt C, series L

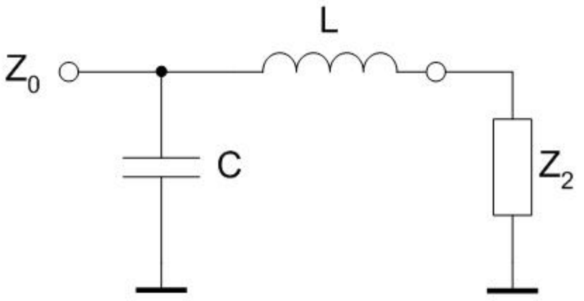




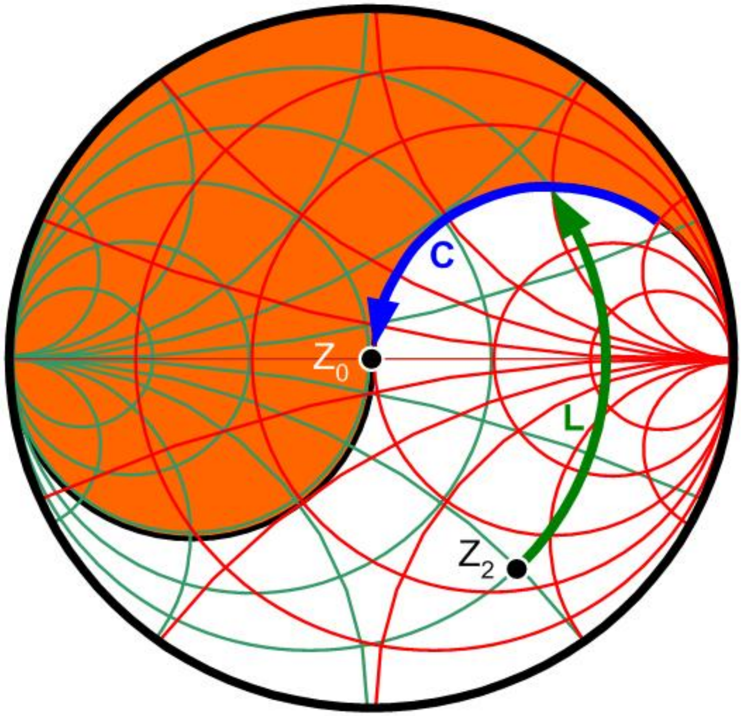


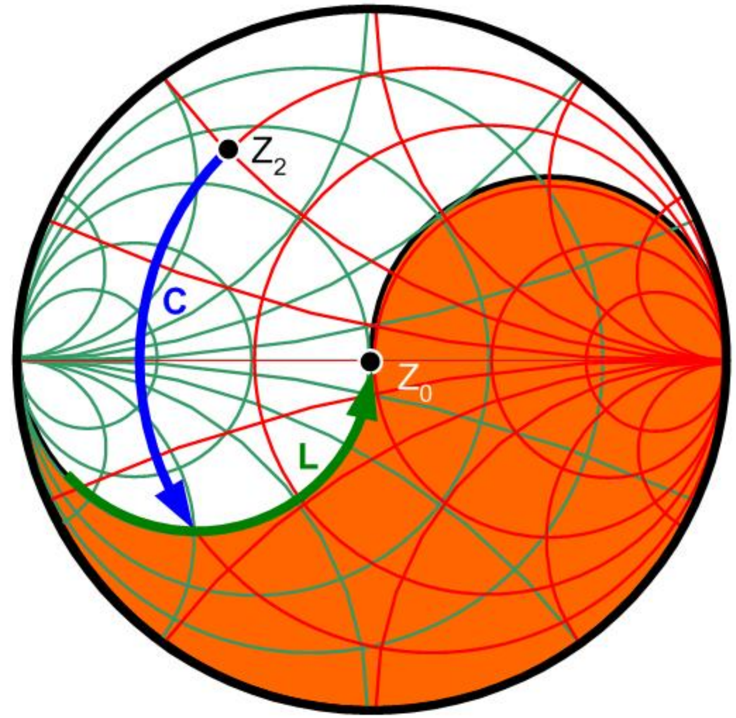


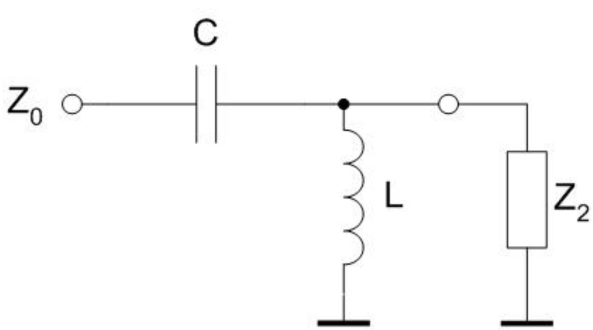
Forbidden area for current network



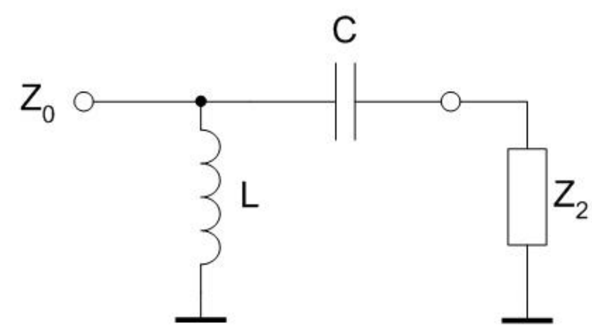
series C, shunt L / shunt L, series C



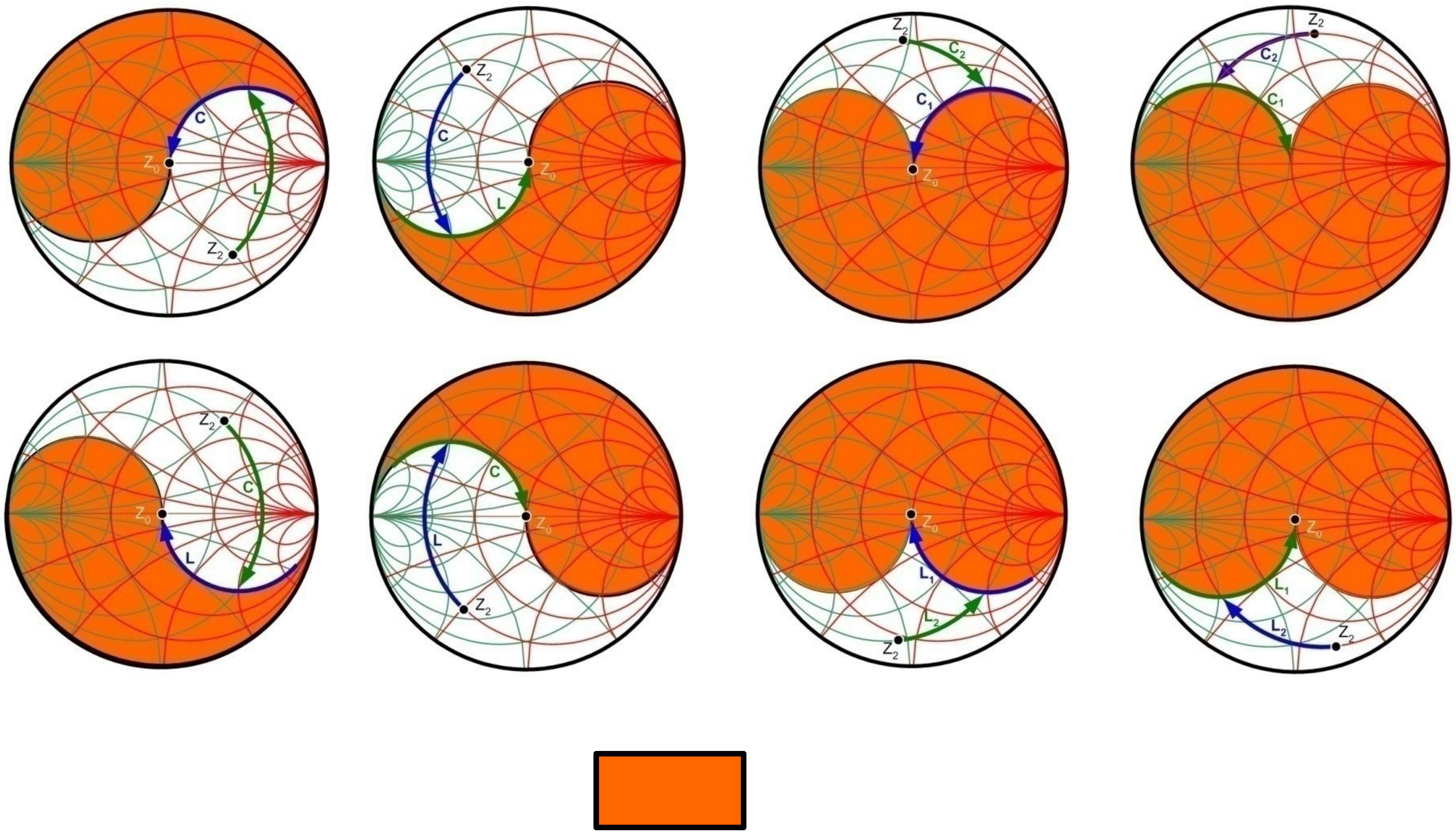




Forbidden area for current network



Matching with 2 reactive elements (L Networks)



Forbidden area for current network

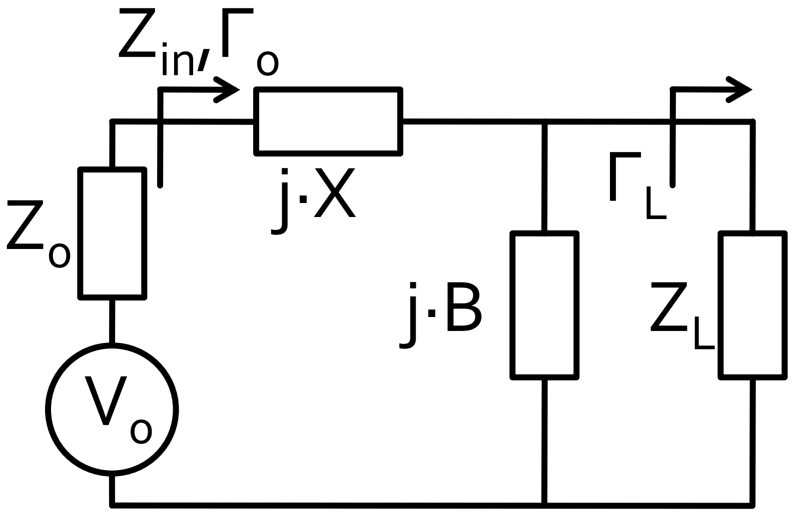
Matching with 2 reactive elements (L Networks)

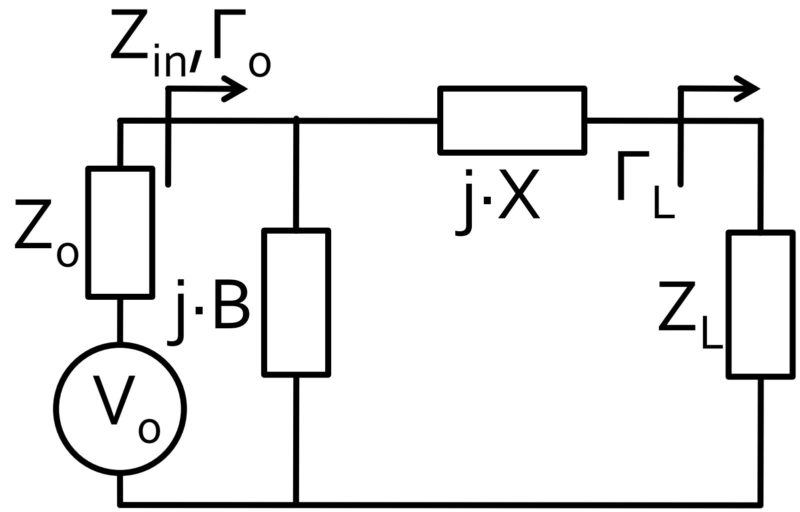
- For any Γ_L there are at least 2 possible L networks to achieve match (L+C)
- For some starting areas on the Smith Chart there are 4 possibilities (+2 C+C/L+L networks)
- We choose the network that requires components with existent/practically realizable values
- By adding the resistive elements, we can supplement the number of networks but with **loss of signal power (not recommended)**

Matching with resistive elements

- In microwave frequencies active circuits work very near to the transition frequency f_T
- Any "waste" of signal power **is not recommended**
- Sometimes such an action might be **necessary** to insure device stability

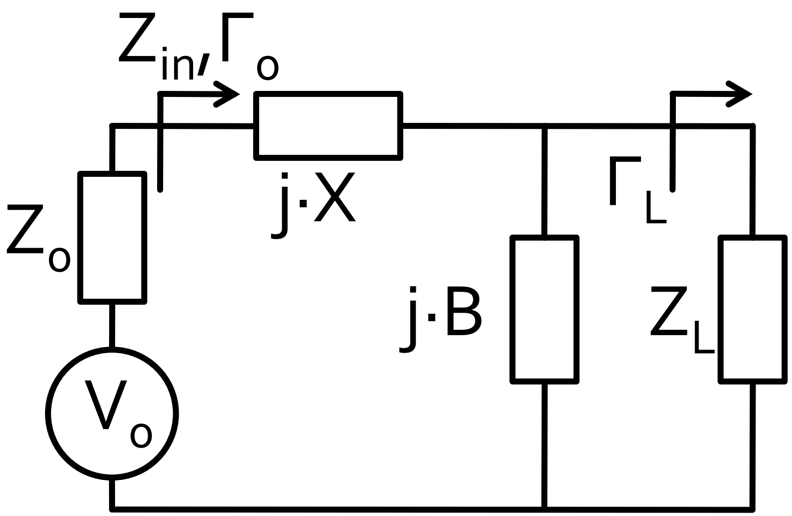
Matching with 2 reactive elements (L Networks)





- Two step matching
  - for starting reflection coefficient inside the $r_L = 1$ circle we must use first schematic
  - for starting reflection coefficient outside the $r_L = 1$ circle we must use second schematic

Matching with 2 reactive elements (L Networks)



$$Z_L = R_L + j \cdot X_L \qquad R_L > Z_0 \qquad Z_{in} = Z_0$$

$$Z_0 = j \cdot X + \frac{1}{j \cdot B + 1/(R_L + j \cdot X_L)}$$

$$\begin{cases} B \cdot (X \cdot R_L - X_L \cdot Z_0) = R_L - Z_0 \\ X \cdot (1 - B \cdot X_L) = B \cdot Z_0 \cdot R_L - X_L \end{cases}$$

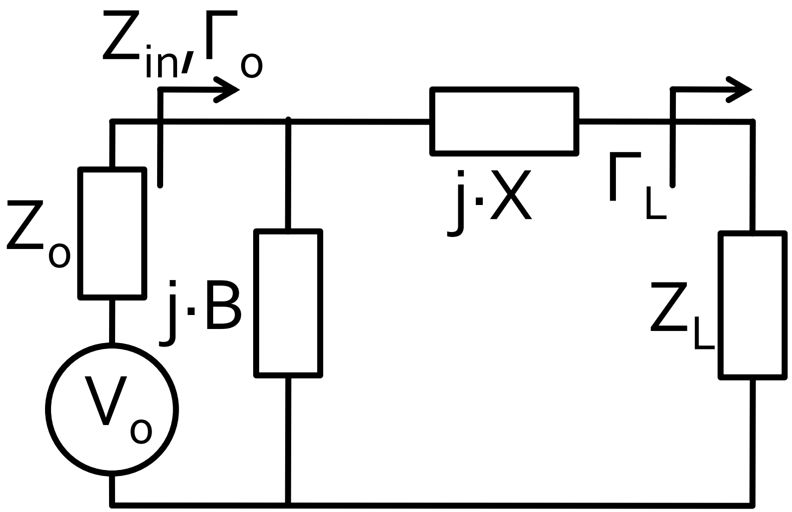
$$B = \frac{X_L \pm \sqrt{R_L / Z_0} \cdot \sqrt{R_L^2 + X_L^2 - Z_0 \cdot R_L}}{R_L^2 + X_L^2} \qquad X = \frac{1}{B} + \frac{X_L \cdot Z_0}{R_L} - \frac{Z_0}{B \cdot R_L}$$

- the argument of the second square root is always positive for:

$$R_L > Z_0$$

- two physically realizable solutions are possible for B and X

Matching with 2 reactive elements (L Networks)



$$Z_L = R_L + j \cdot X_L \qquad R_L < Z_0 \qquad Y_{in} = Y_0 = \frac{1}{Z_0}$$

$$\frac{1}{Z_0} = j \cdot B + \frac{1}{R_L + j \cdot (X + X_L)}$$

$$\begin{cases} B \cdot Z_0 \cdot (X + X_L) = Z_0 - R_L \\ (X + X_L) = B \cdot Z_0 \cdot R_L \end{cases}$$

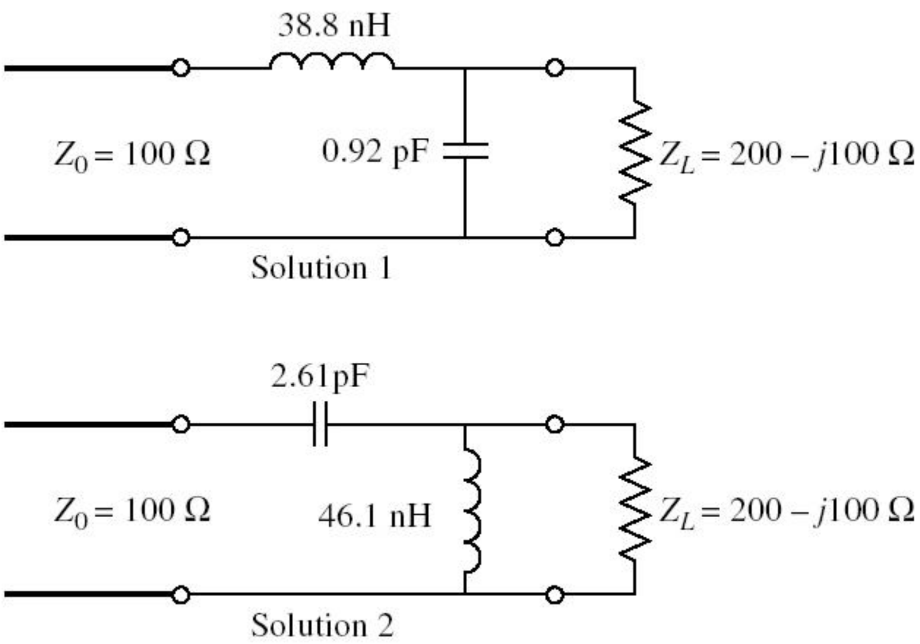
$$X = \pm\sqrt{R_L \cdot (Z_0 - R_L)} - X_L \qquad B = \pm\frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0}$$

- the argument of the square root is always positive for:

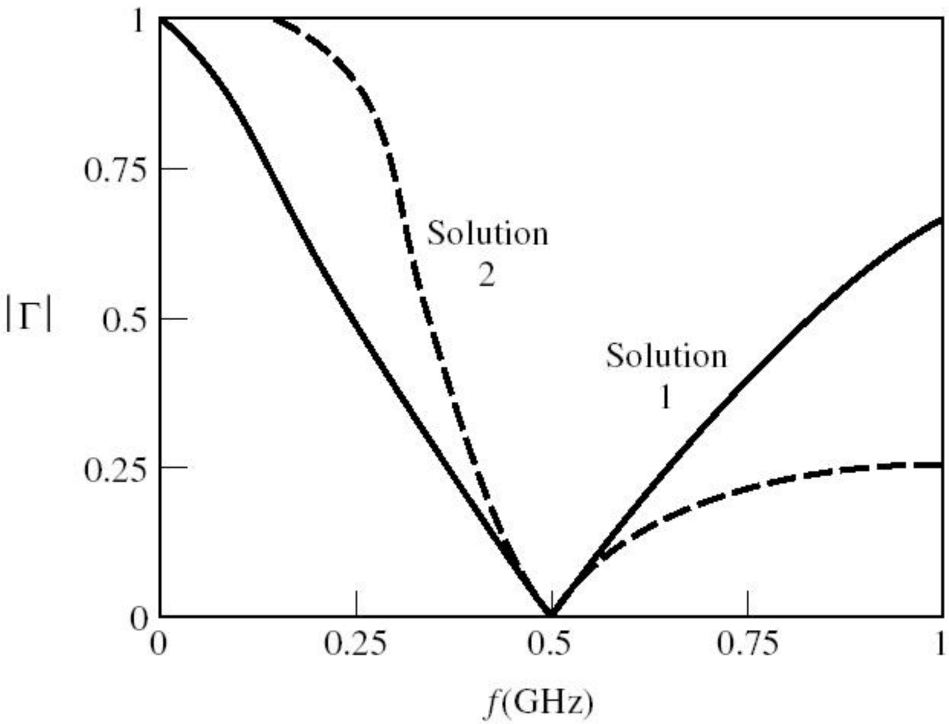
$$R_L < Z_0$$

- two physically realizable solutions are possible for B and X

Example

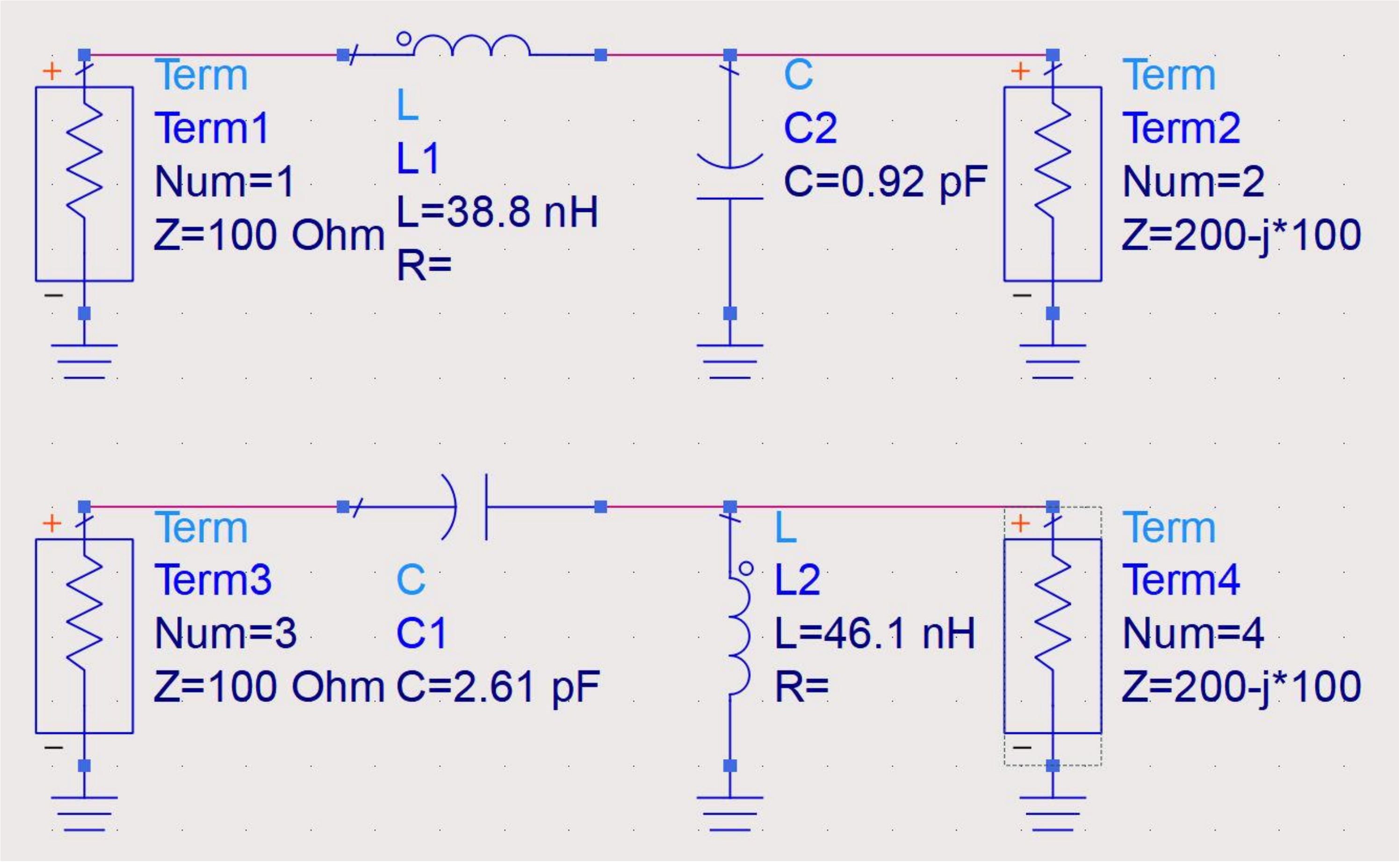


38.8 nH
Z0 = 100 Ω
0.92 pF
ZL = 200 – j100 Ω
Solution 1
2.61pF
Z0 = 100 Ω
46.1 nH
ZL = 200 – j100 Ω
Solution 2



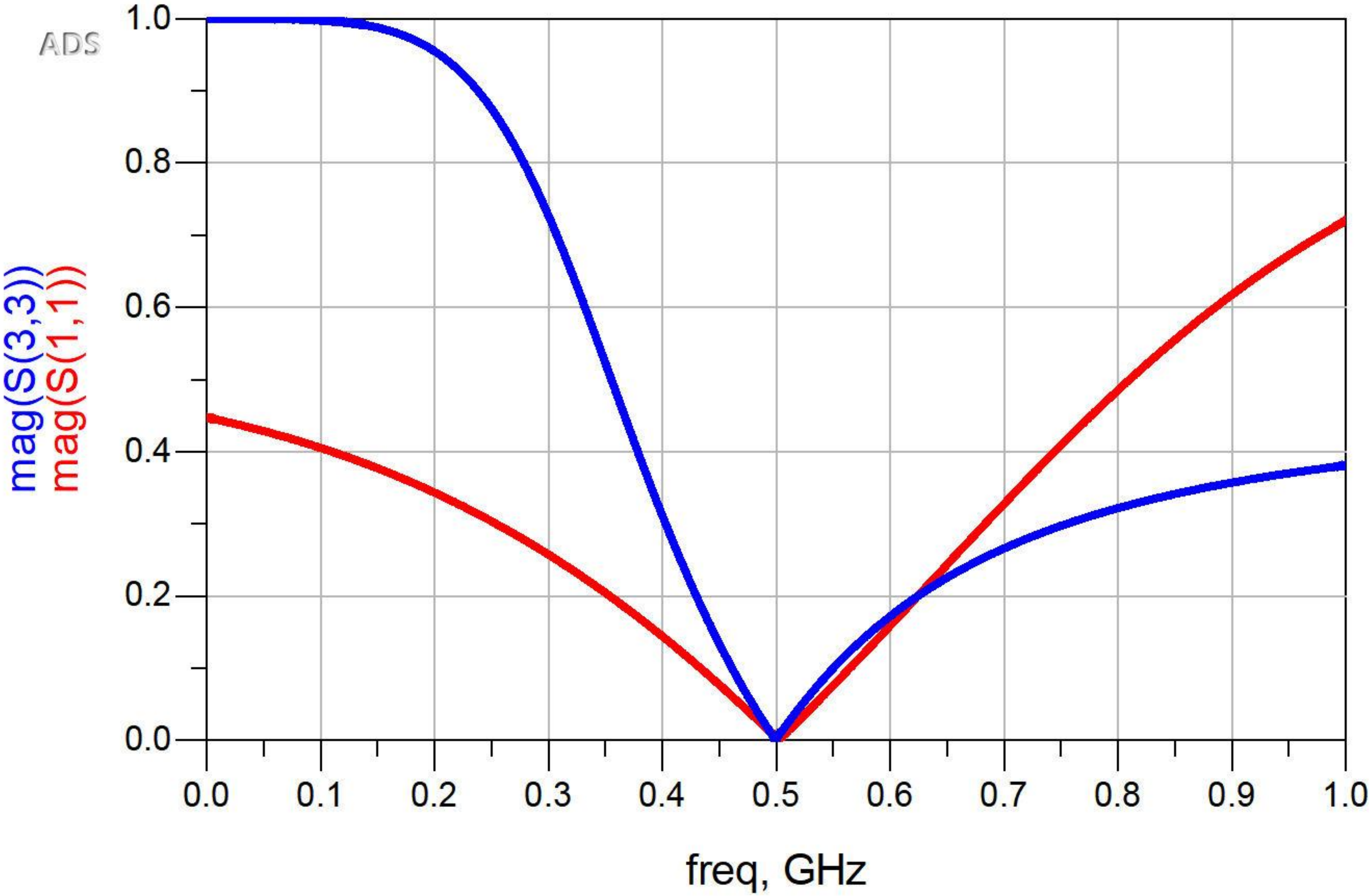
|Γ|
1
0.75
0.5
0.25
0
0
0.25
0.5
0.75
1
f(GHz)
Solution 2
Solution 1

Example



Term
Term1
Num=1
Z=100 Ohm
L
L1
L=38.8 nH
R=
C
C2
C=0.92 pF
Term
Term2
Num=2
Z=200-j*100
Term
Term3
Num=3
Z=100 Ohm
C
C1
C=2.61 pF
L
L2
L=46.1 nH
R=
Term
Term4
Num=4
Z=200-j*100

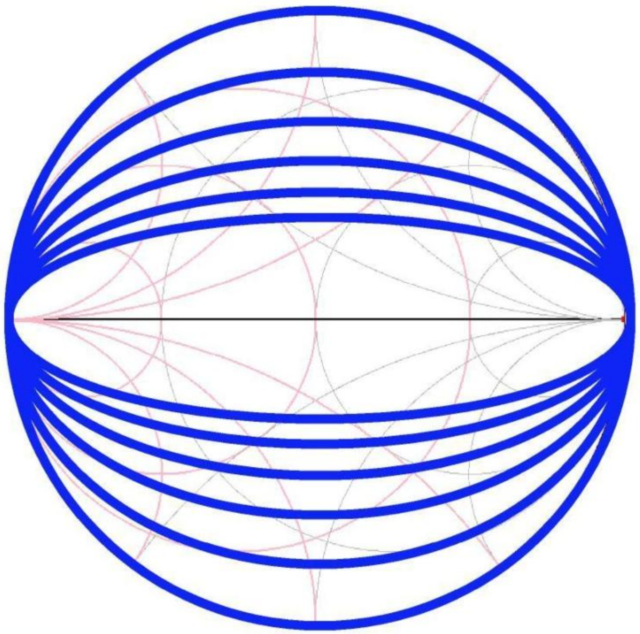
Example

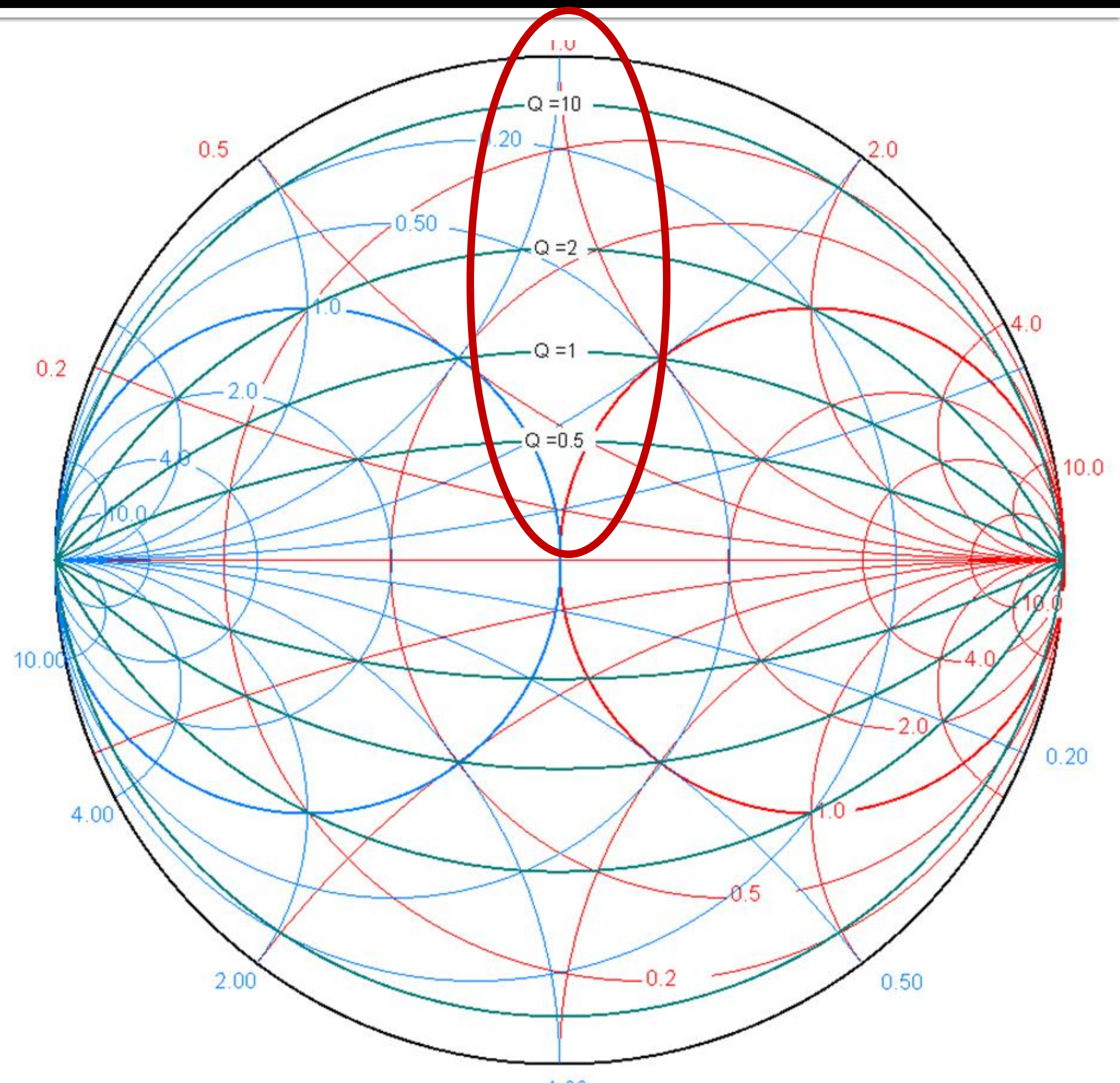

ADS
1.0
0.8
0.6
0.4
0.2
0.0
mag(S(3,3))
mag(S(1,1))
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
freq, GHz

Constant Q circles

- Quality factor Q

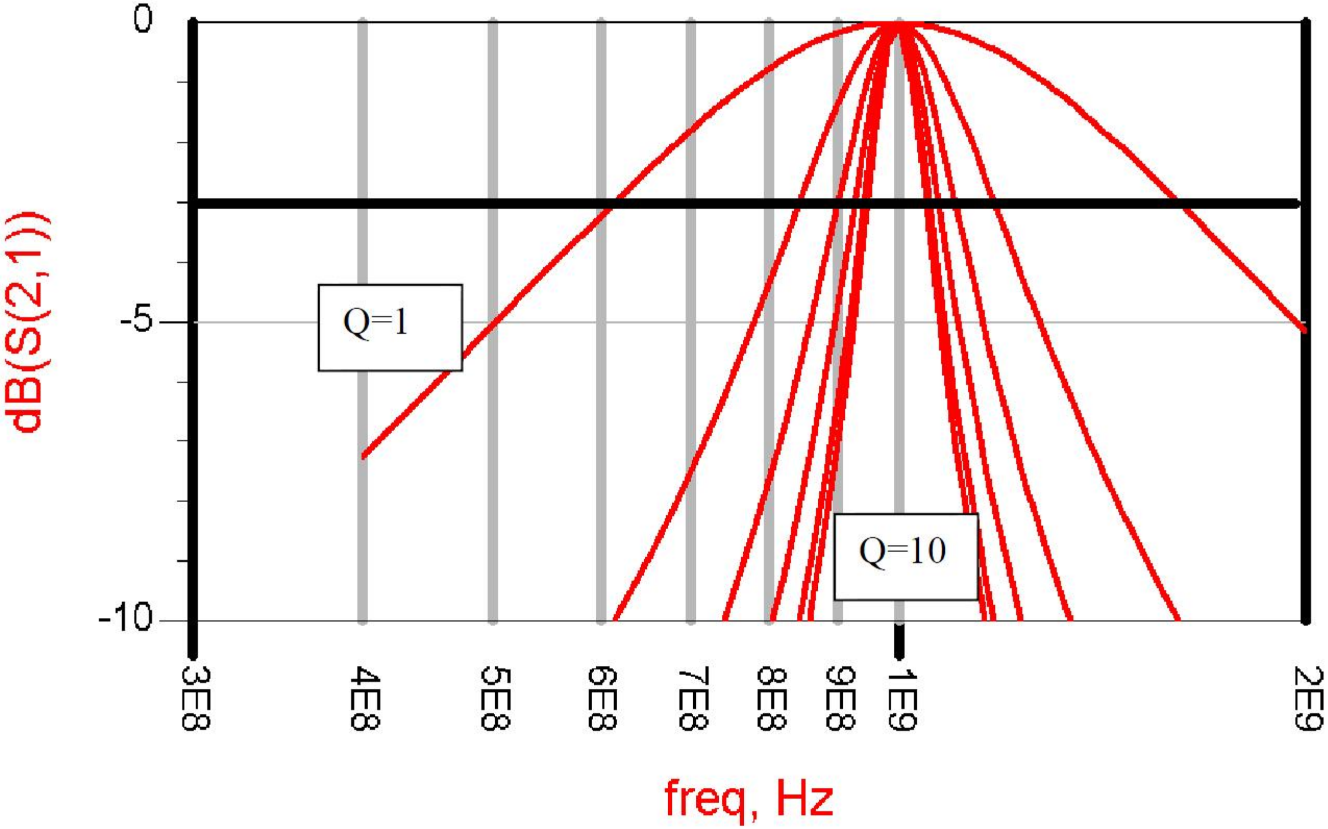
$$Q = \frac{X}{R} = \frac{G}{B} = const$$



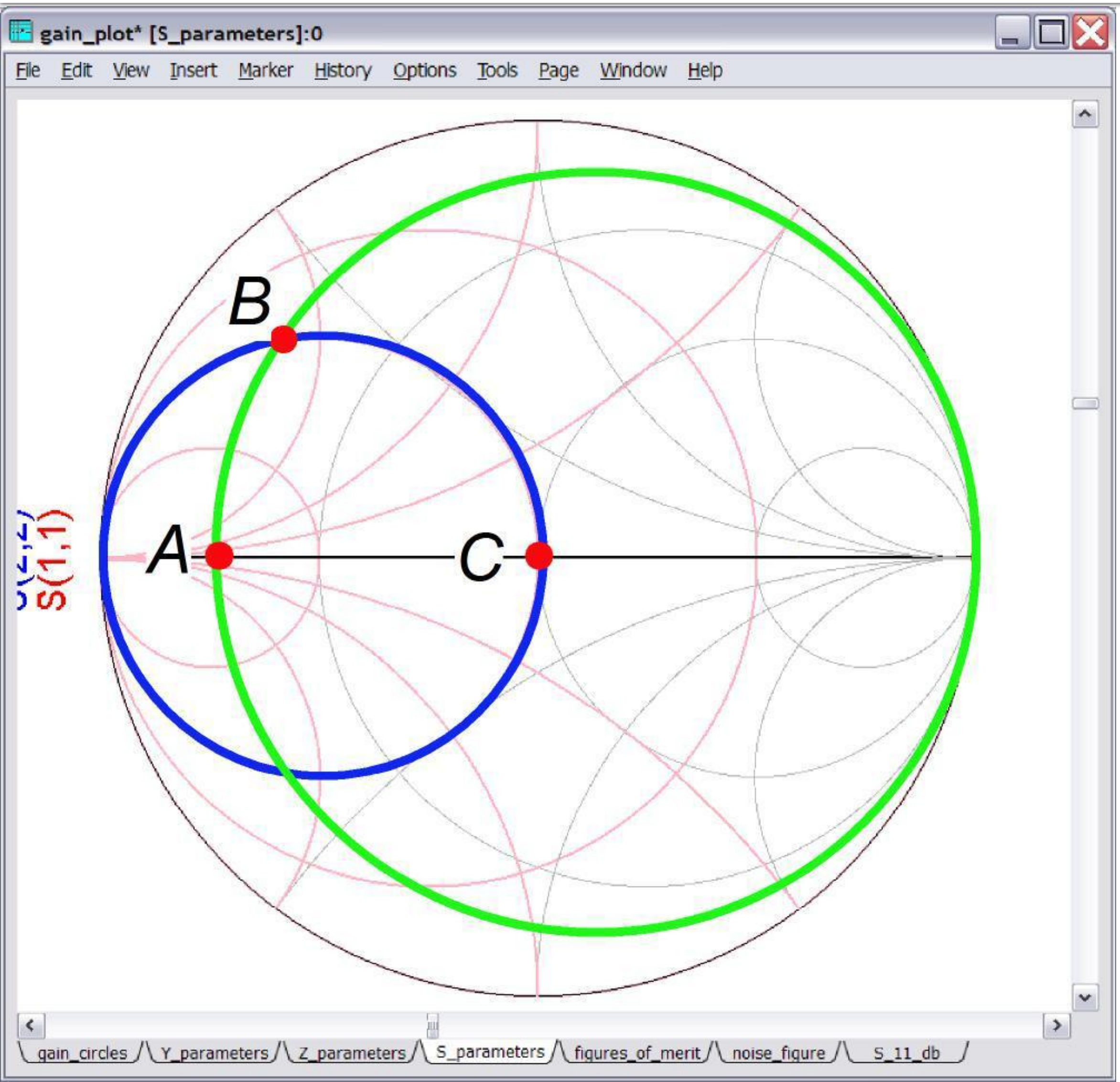


Quality factor - bandwidth

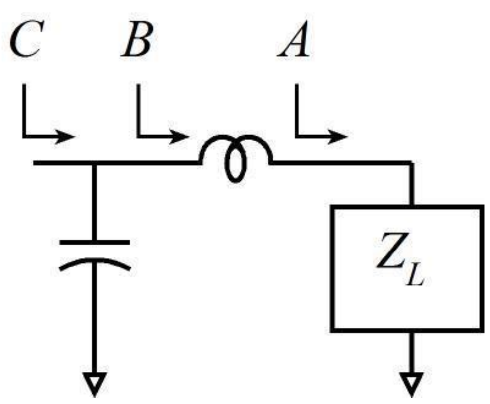
- High quality factor is equivalent with narrow bandwidth



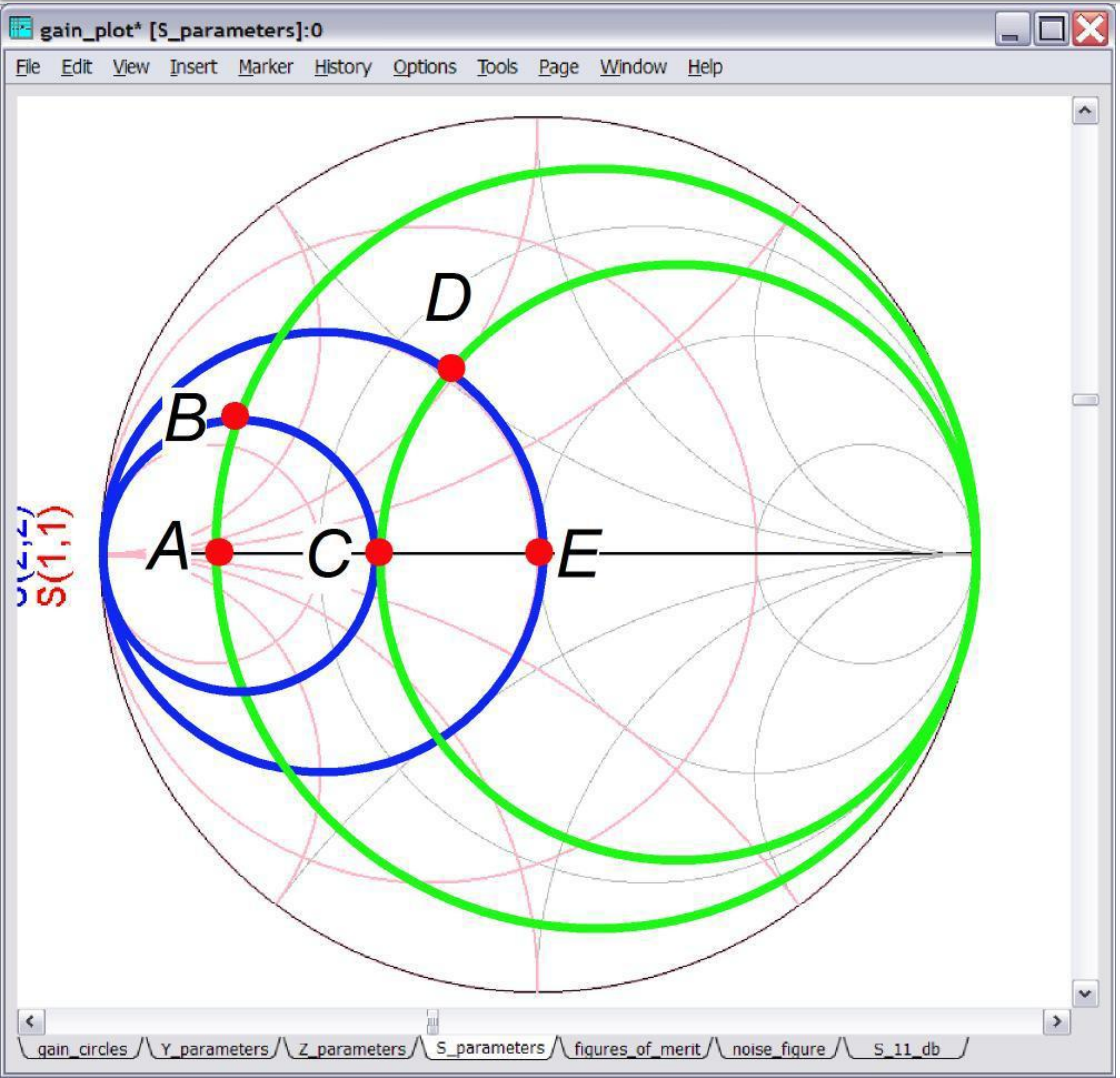
Match bandwidth



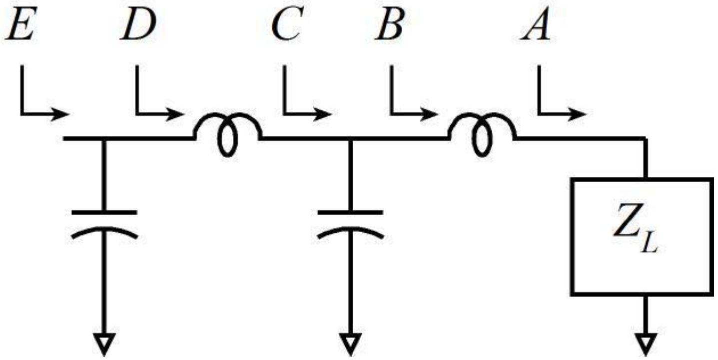
- The position with highest Q for intermediate reflection coefficients (B) imposes the match bandwidth



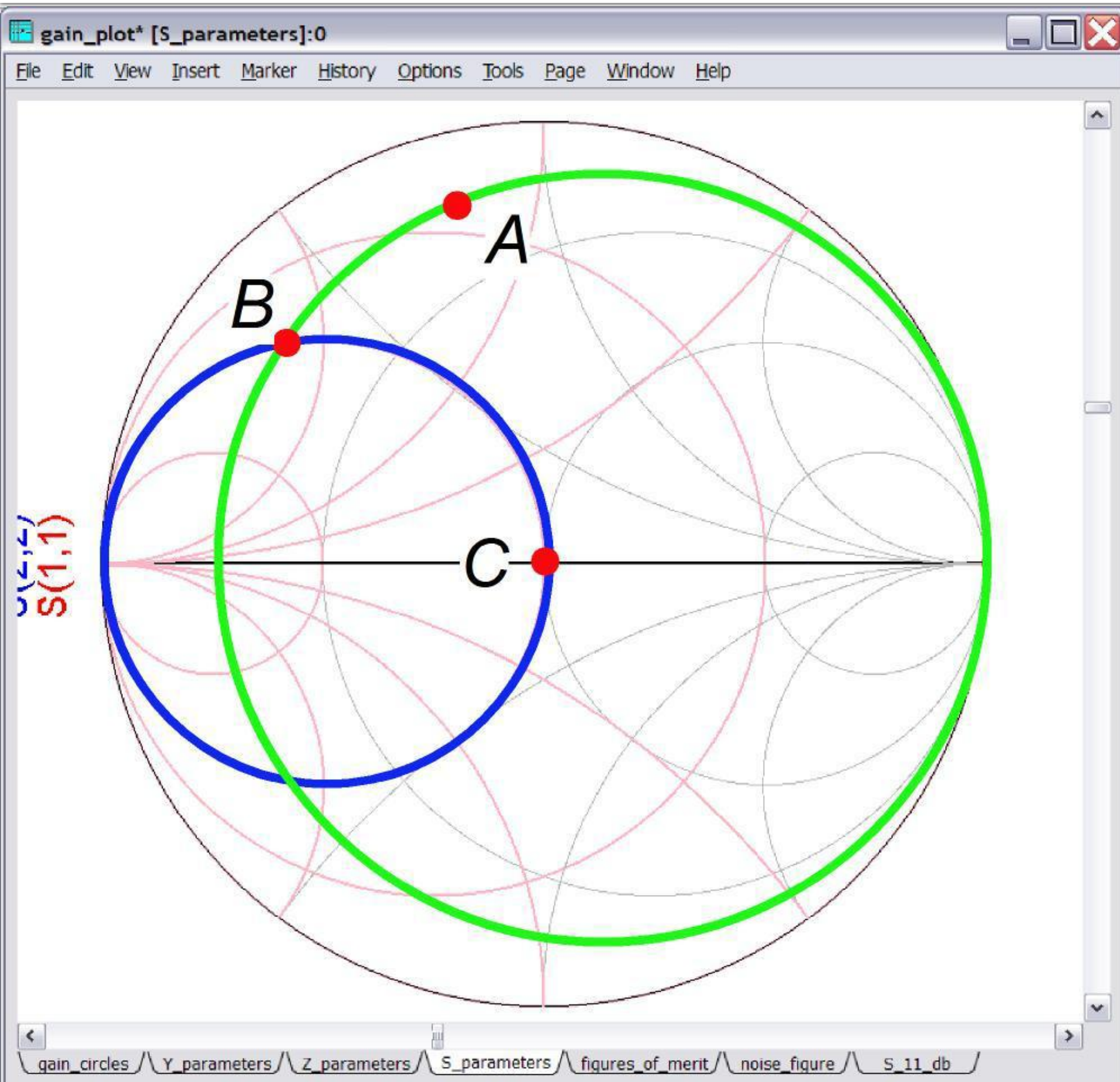
Match bandwidth



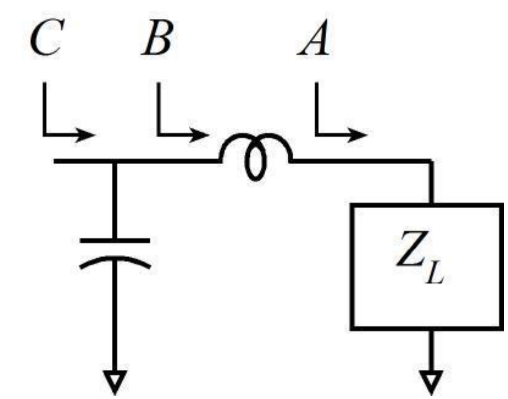
- Wider matching bandwidth can be obtained with multiple, smaller steps, insuring that all intermediate reflection coefficients (B, D) correspond to smaller Q



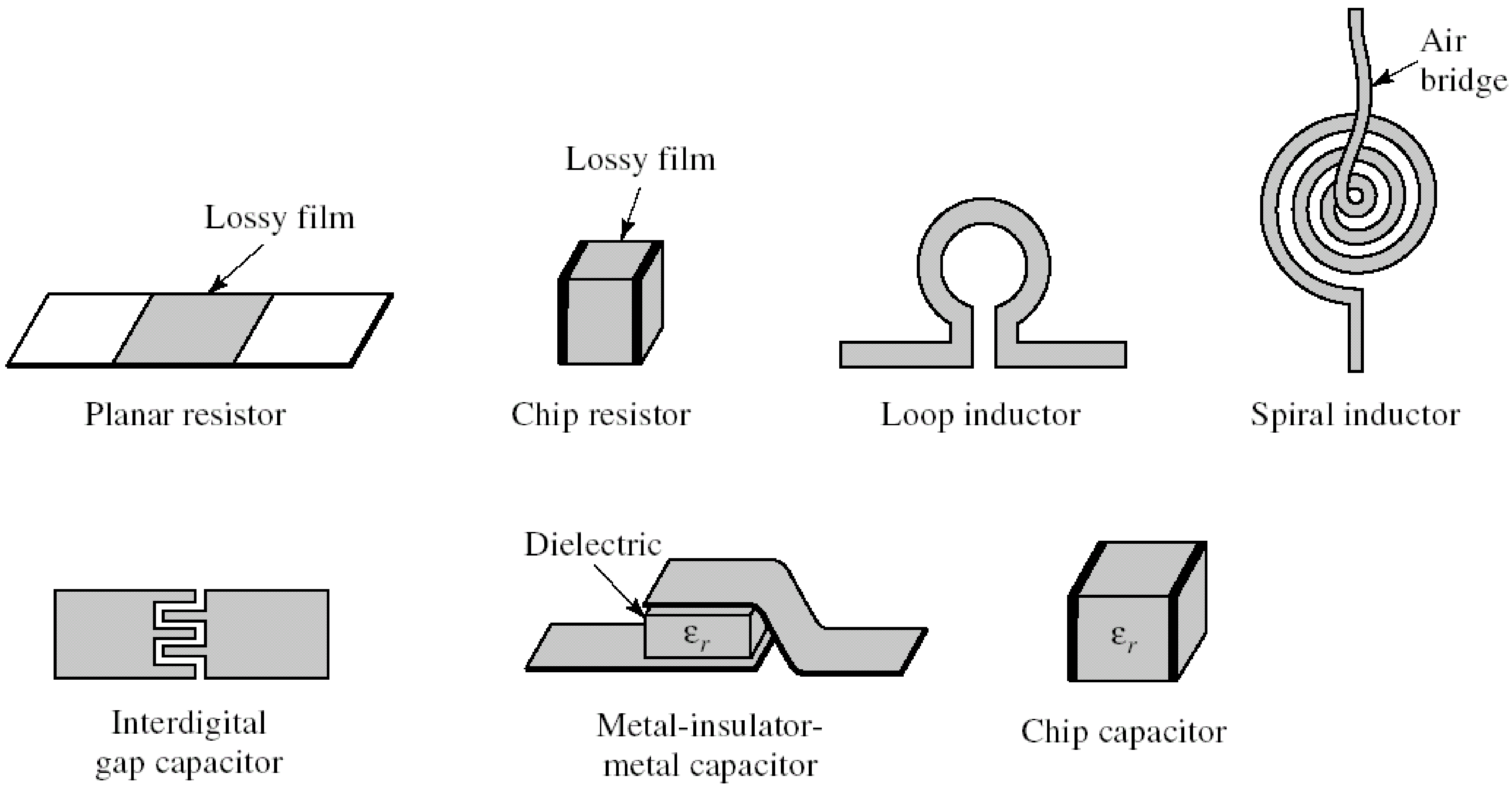
Match bandwidth



- For high Q starting reflection coefficients (A) narrow bandwidth match is unavoidable



Practical realization of lumped elements for microwave frequencies



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